CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam “Wiskunde A”

Date: 16 January 2014

Time: 14.00 – 17.00

Please read the instructions below carefully before answering the questions.

- This exam consists of 7 questions, with in total 22 sub-questions.
- Points that can be scored:

question	1	2	3	4	5	6	7
a	4	4	5	2	3	2	6
b	4	4	6	5	3	5	6
c		2		3	6	2	7
d				2			
e				5			
f				4			
total	8	10	11	21	12	9	19

  You will pass the exam if you score a total of at least 45 points out of a possible 90 points.
- Make sure your name is clearly written on every answer sheet.
- Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your (graphing) calculator are invalid.
- Write your answers in ink. Do not use a pencil, except when drawing graphs.
- You can use a (graphing) calculator. The use of hand-held computers is not allowed. If there is doubt about the status of your equipment, the exam monitor will decide whether it is allowed for use during the exam.
- On page 5 you will find a list of formulas that you may use during this exam. On the last pages of this exam you will find tables of the binomial and standard normal distributions. The use of other formula sheets or books (like BINAS) is not allowed.
- You can use a dictionary if it is approved by the exam monitor.
- Please switch off your mobile telephone.
- Please check www.ccvx.nl for further information on this exam (unfortunately in Dutch only). Answers to the questions will be published on this website next week. Do not call the Open Universiteit, since they do not have any further information about this exam.

1 From an arithmetic sequence the third and the 30st term are given by $u_3 = 15$ and $u_{30} = 123$.

4 pt **a** Is 69 a term in this sequence?
If so, compute the number of this term. In not, explain.

From a geometric sequence the first and the fourth term are given by $u_1 = 1024$ and $u_4 = 3456$.

4 pt **b** Is 59049 a term in this sequence?
If so, compute the number of this term. In not, explain.

2 The total production costs of a certain good are given by the formula

$$C = 27{,}000 \cdot P^{\frac{2}{3}}$$

(C in euro's; P the yearly production in tonnes)

In a given year, the total costs are 432,000 euro.

4 pt **a** Compute the yearly production algebraically.

De derivative function of C is called the marginal cost function (C').

4 pt **b** Find a formula for the marginal cost function and write the answer without negative or broken exponents.

2 pt **c** Use the derivative to compute the marginal costs at a yearly production of 216 tonnes.

3 A window is constructed from a type of glass with the property that a plate of this glass with a thickness of 10 mm absorbs 13% of the light which passes trough. The thickness of the glass in the window is 24 mm.

5 pt **a** Compute the percentage of the light which is absorbed by the glass plate in this window.

At the design of a building, is is important to know the thickness which is required to get a certain transparency of the windows. Therefore a formula is required which gives the thickness of the plate as a function of the percentage light that passes through the window.

6 pt **b** Find a formula which gives the thickness of the glass plate as a function of the percentage of light which passes trough the plate and use this formula to find the thickness of the plate is 40% of the light is absorbed.

4 The Dutch Soccer League consists of 18 clubs, Each club plays a home and an away game against the other 17 clubs. A father bets with his son that his favorite club Ajanoord scores more goals in the 17 home games than in the 17 away games. They decide to compare the home and away games against the same opponent. For example: Ajanoord won the home game against SVP by 3 – 1 and they won the away game against SVP by 1 – 0. Such a comparison results in a plus. If Ajanoord scores less goals home than away against a certain club, a minus is scored and if the same number of goals are scored by Ajanoord in the home and away game against some club, a zero is scored. In the 34 games of the season in which this bet is decided, Ajanoord scores 8 pluses, 2 minuses and 7 zeros.

2 pt **a** State the hypothesis that father and son will test and state the alternative.

5 pt **b** What is the conclusion of this sign test if $\alpha = 0.05$?

This year, the total length of a soccer game, including stoppage time, is normally distributed with an average of 93 minutes and a standard deviation of 1 minute.

3 pt **c** Compute the probability that a random home game of Ajanoord has a length between 92.5 and 94 minutes.

2 pt **d** What is the probability that the home game against SVP had a longer length than the away game against SVP?

The Sports Channel wants to broadcast four randomly selected games from this year's competition.

5 pt **e** Compute the probability that the total length of these four games is less than 6 hours and 7 minutes.

Last year, the length of a soccer game also was 93 minutes. 34% of the games had a length of more than 93.5 minutes. Like this year, the length of te games was normally distributed.

4 pt **f** Compute the standard deviation of the length of a soccer game for last year. Give the answer rounded off to one digit behind the decimal dot.

5 *On 1 November 2013, the Dutch newspaper "De Volkskrant" reported about the good fortune of the 67-year-old mr. James Bozeman from Florida, who won the jackpot of the Lotto for the second time. Both times he selected six random numbers. The newspaper further states that the probability of winning the Lotto is 1 in 14 million. The probability of winning the Lotto for a second time is 1 in 195 billion (U.K.) [= 195 trillion (USA)].*

We assume that the Lotto in Florida is played by randomly drawing six balls from a vase containing 49 balls, numbered from 1 to 49. A participant chooses six from these 49 numbers and wins the jackpot if these six numbers are drawn.

3 pt **a** Use a computation to check that the probability of winning the jackpot is indeed approximately 1 in 14 million.

3 pt **b** What do you think of the statement in the last sentence of the newspaper report?

In the Lotto of Miniland, four numbers are drawn from the numbers 1 to 25. The participant chooses four of these 25 numbers. He wins a price if more than two of his four numbers are drawn.

6 pt **c** Compute the probability that a participant who plays with one ticket, wins no price.

6 In a tidal harbour the height of the water on a certain day is given by the formula

$$H = 350 + 180 \sin(0.5(t - 6.3))$$

In this formula H is the height in cm (from the bottom) and t is the time in hours after midnight.

2 pt **a** Compute algebraically the minimal and the maximal height in cm.

5 pt **b** Compute algebraically the times at which the height of the water is minimal. Give your answer rounded off to minutes.

2 pt **c** Determine the period of H rounded off to minutes.

7 Given the functions $f(x) = \dfrac{10x}{5 + x^2}$, $g(x) = 3 + \sqrt{3 - x}$ and $h(x) = 3 + 2x$.

6 pt **a** Compute the minimal and the maximal value of f algebraically.

6 pt **b** Solve algebraically: $g(x) = h(x)$.

A is the point on the graph of g for which $x_A = 2$.

7 pt **c** Use the derivative to determine an equation for the tangent to the graph of g in point A.

List of formulas for the exam Wiskunde A

Probability

If X and Y are any random variables, then: $\mathrm{E}(X+Y)=\mathrm{E}(X)+\mathrm{E}(Y)$ If furthermore X and Y are independent, then: $\sigma(X+Y)=\sqrt{\sigma^2(X)+\sigma^2(Y)}$
$\sqrt{n}$-law: For n independent repetitions of the same experiment where the result of each experiment is a random variable X, the sum of the results is a random variable S and the mean of the results is a random variable $\overline{X}$. $\mathrm{E}(S)=n\cdot\mathrm{E}(X)$ $\sigma(S)=\sqrt{n}\cdot\sigma(X)$ $\mathrm{E}(\overline{X})=\mathrm{E}(X)$ $\sigma(\overline{X})=\dfrac{\sigma(X)}{\sqrt{n}}$

Binomial Distribution

If X has a binomial distribution with parameters n (number of experiments) and p (probability of succes at each experiment), then:

$$P(X=k)=\binom{n}{k}\cdot p^k\cdot(1-p)^{n-k} \quad \text{with } k=0,1,2,\ \ldots\ ,n$$

Expectation: $\mathrm{E}(X)=n\cdot p$ Standard deviation: $\sigma(X)=\sqrt{n\cdot p\cdot(1-p)}$

Normal Distribution

If X is a normally distributed random variable with mean μ and standard deviation σ, then:

$Z=\dfrac{X-\mu}{\sigma}$ has the standard normal distribution and $P(X<g)=P\left(Z<\dfrac{g-\mu}{\sigma}\right)$

Differentiation

rule	function	derivative function
Sum rule	$s(x)=f(x)+g(x)$	$s'(x)=f'(x)+g'(x)$
Product rule	$p(x)=f(x)\cdot g(x)$	$p'(x)=f'(x)\cdot g(x)+f(x)\cdot g'(x)$
Quotient rule	$q(x)=\dfrac{f(x)}{g(x)}$	$q'(x)=\dfrac{f'(x)\cdot g(x)-f(x)\cdot g'(x)}{(g(x))^2}$
Chain rule	$k(x)=f(g(x))$	$k'(x)=f'(g(x))\cdot g'(x)$ or $\dfrac{dk}{dx}=\dfrac{df}{dg}\cdot\dfrac{dg}{dx}$

Logarithms

rule	conditions
$\log_g a+\log_g b=\log_g ab$	$g>0,\ g\neq 1,\ a>0,\ b>0$
$\log_g a-\log_g b=\log_g \dfrac{a}{b}$	$g>0,\ g\neq 1,\ a>0,\ b>0$
$\log_g a^p=p\cdot\log_g a$	$g>0,\ g\neq 1,\ a>0$
$\log_g a=\dfrac{\log_p a}{\log_p g}$	$g>0,\ g\neq 1,\ a>0,\ p>0,\ p\neq 1$

Arithmetic and geometric sequences

Arithmetic sequence:	Sum $=\frac{1}{2}\cdot$ number of terms $\cdot\ (u_e+u_l)$
Geometric sequence:	Sum $=\dfrac{u_{l+1}-u_e}{r-1}$ $(r\neq 1)$
In both formulas	e = number first term of the sum; l = number last term of the sum.

CUMULATIEVE BINOMIALE VERDELINGEN (vervolg)

Verdelingsfunctie $B_{n,p}(x) = P(X \leq x) = \sum_{k=0}^{k=x} \binom{n}{k} p^k (1-p)^{n-k}$

CUMULATIEVE BINOMIALE VERDELINGEN (vervolg)

n	x	p = 0,0500	0,1000	0,1500	0,2000	0,2500	0,3000	0,3500	0,4000	0,4500	0,5000	0,5500	0,6000	0,6500	0,7000	0,7500	0,8000	0,8500	0,9000	0,9500	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{5}{6}$
10	0	0,5987	0,3487	0,1969	0,1074	0,0563	0,0282	0,0135	0,0060	0,0025	0,0010	0,0003	0,0001	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,1615	0,0173	0,0000	0,0000
	1	0,9139	0,7361	0,5443	0,3758	0,2440	0,1493	0,0860	0,0464	0,0233	0,0107	0,0045	0,0017	0,0005	0,0001	0,0000	0,0000	0,0000	0,0000	0,0000	0,4845	0,1040	0,0004	0,0000
	2	0,9885	0,9298	0,8202	0,6778	0,5256	0,3828	0,2616	0,1673	0,0996	0,0547	0,0274	0,0123	0,0048	0,0016	0,0004	0,0001	0,0000	0,0000	0,0000	0,7752	0,2991	0,0034	0,0000
	3	0,9990	0,9872	0,9500	0,8791	0,7759	0,6496	0,5138	0,3823	0,2660	0,1719	0,1020	0,0548	0,0260	0,0106	0,0035	0,0009	0,0001	0,0000	0,0000	0,9303	0,5593	0,0197	0,0003
	4	0,9999	0,9984	0,9901	0,9672	0,9219	0,8497	0,7515	0,6331	0,5044	0,3770	0,2616	0,1662	0,0949	0,0473	0,0197	0,0064	0,0014	0,0001	0,0000	0,9845	0,7869	0,0766	0,0024
	5	1,0000	0,9999	0,9986	0,9936	0,9803	0,9527	0,9051	0,8338	0,7384	0,6230	0,4956	0,3669	0,2485	0,1503	0,0781	0,0328	0,0099	0,0016	0,0001	0,9976	0,9234	0,2131	0,0155
	6	1,0000	1,0000	0,9999	0,9991	0,9965	0,9894	0,9740	0,9452	0,8980	0,8281	0,7340	0,6177	0,4862	0,3504	0,2241	0,1209	0,0500	0,0128	0,0010	0,9997	0,9803	0,4407	0,0697
	7	1,0000	1,0000	1,0000	0,9999	0,9996	0,9984	0,9952	0,9877	0,9726	0,9453	0,9004	0,8327	0,7384	0,6172	0,4744	0,3222	0,1798	0,0702	0,0115	1,0000	0,9966	0,7009	0,2248
	8	1,0000	1,0000	1,0000	1,0000	1,0000	0,9999	0,9995	0,9983	0,9955	0,9893	0,9767	0,9536	0,9140	0,8507	0,7560	0,6242	0,4557	0,2639	0,0861	1,0000	0,9996	0,8960	0,5155
	9	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9999	0,9997	0,9990	0,9975	0,9940	0,9865	0,9718	0,9437	0,8926	0,8031	0,6513	0,4013	1,0000	1,0000	0,9827	0,8385
	10	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000

CUMULATIEVE BINOMIALE VERDELINGEN (vervolg)

Verdelingsfunctie $B_{n,p}(x) = P(X \leq x) = \sum_{k=0}^{k=x} \binom{n}{k} p^k (1-p)^{n-k}$

CUMULATIEVE BINOMIALE VERDELINGEN (vervolg)

n	x	p = 0,0500	0,1000	0,1500	0,2000	0,2500	0,3000	0,3500	0,4000	0,4500	0,5000	0,5500	0,6000	0,6500	0,7000	0,7500	0,8000	0,8500	0,9000	0,9500	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{5}{6}$
20	0	0,3585	0,1216	0,0388	0,0115	0,0032	0,0008	0,0002	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0261	0,0003	0,0000	0,0000
	1	0,7358	0,3917	0,1756	0,0692	0,0243	0,0076	0,0021	0,0005	0,0001	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,1304	0,0033	0,0000	0,0000
	2	0,9245	0,6769	0,4049	0,2061	0,0913	0,0355	0,0121	0,0036	0,0009	0,0002	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,3287	0,0176	0,0000	0,0000
	3	0,9841	0,8670	0,6477	0,4114	0,2252	0,1071	0,0444	0,0160	0,0049	0,0013	0,0003	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,5665	0,0604	0,0000	0,0000
	4	0,9974	0,9568	0,8298	0,6296	0,4148	0,2375	0,1182	0,0510	0,0189	0,0059	0,0015	0,0003	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,7687	0,1515	0,0000	0,0000
	5	0,9997	0,9887	0,9327	0,8042	0,6172	0,4164	0,2454	0,1256	0,0553	0,0207	0,0064	0,0016	0,0003	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,8982	0,2972	0,0002	0,0000
	6	1,0000	0,9976	0,9781	0,9133	0,7858	0,6080	0,4166	0,2500	0,1299	0,0577	0,0214	0,0065	0,0015	0,0003	0,0000	0,0000	0,0000	0,0000	0,0000	0,9629	0,4793	0,0009	0,0000
	7	1,0000	0,9996	0,9941	0,9679	0,8982	0,7723	0,6010	0,4159	0,2520	0,1316	0,0580	0,0210	0,0060	0,0013	0,0002	0,0000	0,0000	0,0000	0,0000	0,9887	0,6615	0,0037	0,0000
	8	1,0000	0,9999	0,9987	0,9900	0,9591	0,8867	0,7624	0,5956	0,4143	0,2517	0,1308	0,0565	0,0196	0,0051	0,0009	0,0001	0,0000	0,0000	0,0000	0,9972	0,8095	0,0130	0,0000
	9	1,0000	1,0000	0,9998	0,9974	0,9861	0,9520	0,8782	0,7553	0,5914	0,4119	0,2493	0,1275	0,0532	0,0171	0,0039	0,0006	0,0000	0,0000	0,0000	0,9994	0,9081	0,0376	0,0001
	10	1,0000	1,0000	1,0000	0,9994	0,9961	0,9829	0,9468	0,8725	0,7507	0,5881	0,4086	0,2447	0,1218	0,0480	0,0139	0,0026	0,0002	0,0000	0,0000	0,9999	0,9624	0,0919	0,0006
	11	1,0000	1,0000	1,0000	0,9999	0,9991	0,9949	0,9804	0,9435	0,8692	0,7483	0,5857	0,4044	0,2376	0,1133	0,0409	0,0100	0,0013	0,0001	0,0000	1,0000	0,9870	0,1905	0,0028
	12	1,0000	1,0000	1,0000	1,0000	0,9998	0,9987	0,9940	0,9790	0,9420	0,8684	0,7480	0,5841	0,3990	0,2277	0,1018	0,0321	0,0059	0,0004	0,0000	1,0000	0,9963	0,3385	0,0113
	13	1,0000	1,0000	1,0000	1,0000	1,0000	0,9997	0,9985	0,9935	0,9786	0,9423	0,8701	0,7500	0,5834	0,3920	0,2142	0,0867	0,0219	0,0024	0,0000	1,0000	0,9991	0,5207	0,0371
	14	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9997	0,9984	0,9936	0,9793	0,9447	0,8744	0,7546	0,5836	0,3828	0,1958	0,0673	0,0113	0,0003	1,0000	0,9998	0,7028	0,1018
	15	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9997	0,9985	0,9941	0,9811	0,9490	0,8818	0,7625	0,5852	0,3704	0,1702	0,0432	0,0026	1,0000	1,0000	0,8485	0,2313
	16	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9997	0,9987	0,9951	0,9840	0,9556	0,8929	0,7748	0,5886	0,3523	0,1330	0,0159	1,0000	1,0000	0,9396	0,4335
	17	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9998	0,9991	0,9964	0,9879	0,9645	0,9087	0,7939	0,5951	0,3231	0,0755	1,0000	1,0000	0,9824	0,6713
	18	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9999	0,9995	0,9979	0,9924	0,9757	0,9308	0,8244	0,6083	0,2642	1,0000	1,0000	0,9967	0,8696
	19	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9998	0,9992	0,9968	0,9885	0,9612	0,8784	0,6415	1,0000	1,0000	0,9997	0,9739
	20	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000

CUMULATIEVE BINOMIALE VERDELINGEN (vervolg)

Verdelingsfunctie $B_{n,p}(x) = P(X \leq x) = \sum_{k=0}^{k=x} \binom{n}{k} p^k (1-p)^{n-k}$

n	x	p = 0,0100	0,0500	0,1000	0,1500	0,2000	0,2500	0,3000	0,3500	0,4000	0,4500	0,5000
50	0	0,6050	0,0769	0,0052	0,0003	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
	1	0,9106	0,2794	0,0338	0,0029	0,0002	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
	2	0,9862	0,5405	0,1117	0,0142	0,0013	0,0001	0,0000	0,0000	0,0000	0,0000	0,0000
	3	0,9984	0,7604	0,2503	0,0460	0,0057	0,0005	0,0000	0,0000	0,0000	0,0000	0,0000
	4	0,9999	0,8964	0,4312	0,1121	0,0185	0,0021	0,0002	0,0000	0,0000	0,0000	0,0000
	5	1,0000	0,9622	0,6161	0,2194	0,0480	0,0070	0,0007	0,0001	0,0000	0,0000	0,0000
	6	1,0000	0,9882	0,7702	0,3613	0,1034	0,0194	0,0025	0,0002	0,0000	0,0000	0,0000
	7	1,0000	0,9968	0,8779	0,5188	0 1904	0,0453	0,0073	0,0008	0,0001	0,0000	0,0000
	8	1,0000	0,9992	0,9421	0,6681	0,3073	0,0916	0,0183	0 0025	0,0002	0,0000	0,0000
	9	1,0000	0,9998	0,9755	0,7911	0,4437	0,1637	0,0402	0,0067	0,0008	0,0001	0,0000
	10	1,0000	1,0000	0,9906	0,8801	0,5836	0,2622	0,0789	0,0160	0,0022	0,0002	0,0000
	11	1,0000	1,0000	0,9968	0,9372	0,7107	0,3816	0,1390	0,0342	0,0057	0,0006	0,0000
	12	1,0000	1,0000	0,9990	0,9699	0,8139	0,5110	0,2229	0,0661	0,0133	0,0018	0,0002
	13	1,0000	1,0000	0,9997	0,9868	0,8894	0,6370	0,3279	0,1163	0,0280	0,0045	0,0005
	14	1,0000	1,0000	0,9999	0,9947	0,9393	0,7481	0,4468	0,1878	0,0540	0,0104	0,0013
	15	1,0000	1,0000	1,0000	0,9981	0,9692	0,8369	0,5692	0,2801	0,0955	0,0220	0,0033
	16	1,0000	1,0000	1,0000	0,9993	0,9856	0,9017	0,6839	0,3889	0,1561	0,0427	0,0077
	17	1,0000	1,0000	1,0000	0,9998	0,9937	0,9449	0,7822	0,5060	0,2369	0,0765	0,0164
	18	1,0000	1,0000	1,0000	0,9999	0,9975	0,9713	0,8594	0,6216	0,3356	0,1273	0,0325
	19	1,0000	1,0000	1,0000	1,0000	0,9991	0,9861	0,9152	0,7264	0,4465	0,1974	0,0595
	20	1,0000	1,0000	1,0000	1,0000	0,9997	0,9937	0,9522	0,8139	0,5610	0,2862	0,1013
	21	1,0000	1,0000	1,0000	1,0000	0,9999	0,9974	0,9749	0,8813	0,6701	0,3900	0,1611
	22	1,0000	1,0000	1,0000	1,0000	1,0000	0,9990	0,9877	0,9290	0,7660	0,5019	0,2399
	23	1,0000	1,0000	1,0000	1,0000	1,0000	0,9996	0,9944	0,9604	0,8438	0,6134	0,3359
	24	1,0000	1,0000	1,0000	1,0000	1,0000	0,9999	0,9976	0,9793	0,9022	0,7160	0,4439
	25	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9991	0,9900	0,9427	0,8034	0,5561
	26	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9997	0,9955	0,9686	0,8721	0,6641
	27	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9999	0,9981	0,9840	0,9220	0,7601
	28	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9993	0,9924	0,9556	0,8389
	29	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9997	0,9966	0,9765	0,8987
	30	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9999	0,9986	0,9884	0,9405
	31	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9995	0,9947	0,9675
	32	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9998	0,9978	0,9836
	33	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9999	0,9991	0,9923
	34	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9997	0,9967
	35	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9999	0,9987
	36	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9995
	37	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9998
	38	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	39	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	40	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	41	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	42	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	43	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	44	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	45	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	46	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	47	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	48	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	49	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	50	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000

CUMULATIEVE BINOMIALE VERDELINGEN (vervolg)

n	x	p = 0,5500	0,6000	0,6500	0,7000	0,7500	0,8000	0,8500	0,9000	0,9500	0,9900	1/6	1/3	2/3	5/6
50	0	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0000
	1	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0012	0,0000	0,0000	0,0000
	2	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0066	0,0000	0,0000	0,0000
	3	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0238	0,0000	0,0000	0,0000
	4	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0643	0,0000	0,0000	0,0000
	5	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,1388	0,0001	0,0000	0,0000
	6	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,2506	0,0005	0,0000	0,0000
	7	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,3911	0,0017	0,0000	0,0000
	8	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,5421	0,0050	0,0000	0,0000
	9	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,6830	0,0127	0,0000	0,0000
	10	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,7986	0,0284	0,0000	0,0000
	11	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,8827	0,0570	0,0000	0,0000
	12	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,9373	0,1035	0,0000	0,0000
	13	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,9693	0,1715	0,0000	0,0000
	14	0,0001	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,9862	0,2612	0,0000	0,0000
	15	0,0003	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,9943	0,3690	0,0000	0,0000
	16	0,0009	0,0001	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,9978	0,4868	0,0000	0,0000
	17	0,0022	0,0002	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,9992	0,6046	0,0000	0,0000
	18	0,0053	0,0005	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,9997	0,7126	0,0000	0,0000
	19	0,0116	0,0014	0,0001	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,9999	0,8036	0,0000	0,0000
	20	0,0235	0,0034	0,0003	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	1,0000	0,8741	0,0001	0,0000
	21	0,0444	0,0076	0,0007	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	1,0000	0,9244	0,0003	0,0000
	22	0,0780	0,0160	0,0019	0,0001	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	1,0000	0,9576	0,0008	0,0000
	23	0,1279	0,0314	0,0045	0,0003	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	1,0000	0,9778	0,0021	0,0000
	24	0,1966	0,0573	0,0100	0,0009	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	1,0000	0,9892	0,0049	0,0000
	25	0,2840	0,0978	0,0207	0,0024	0,0001	0,0000	0,0000	0,0000	0,0000	0,0000	1,0000	0,9951	0,0108	0,0000
	26	0,3866	0,1562	0,0396	0,0056	0,0004	0,0000	0,0000	0,0000	0,0000	0,0000	1,0000	0,9979	0,0222	0,0000
	27	0,4981	0,2340	0,0710	0,0123	0,0010	0,0000	0,0000	0,0000	0,0000	0,0000	1,0000	0,9992	0,0424	0,0000
	28	0,6100	0,3299	0,1187	0,0251	0,0026	0,0001	0,0000	0,0000	0,0000	0,0000	1,0000	0,9997	0,0756	0,0000
	29	0,7138	0,4390	0,1861	0,0478	0,0063	0,0003	0,0000	0,0000	0,0000	0,0000	1,0000	0,9999	0,1259	0,0000
	30	0,8026	0,5535	0,2736	0,0848	0,0139	0,0009	0,0000	0,0000	0,0000	0,0000	1,0000	1,0000	0,1964	0,0001
	31	0,8727	0,6644	0,3784	0,1406	0,0287	0,0025	0,0001	0,0000	0,0000	0,0000	1,0000	1,0000	0,2874	0,0003
	32	0,9235	0,7631	0,4940	0,2178	0,0551	0,0063	0,0002	0,0000	0,0000	0,0000	1,0000	1,0000	0,3954	0,0008
	33	0,9573	0,8439	0,6111	0,3161	0,0983	0,0144	0,0007	0,0000	0,0000	0,0000	1,0000	1,0000	0,5132	0,0022
	34	0,9780	0,9045	0,7199	0,4308	0,1631	0,0308	0,0019	0,0000	0,0000	0,0000	1,0000	1,0000	0,6310	0,0057
	35	0,9896	0,9460	0,8122	0,5532	0,2519	0,0607	0,0053	0,0001	0,0000	0,0000	1,0000	1,0000	0,7388	0,0138
	36	0,9955	0,9720	0,8837	0,6721	0,3630	0,1106	0,0132	0,0003	0,0000	0,0000	1,0000	1,0000	0,8285	0,0307
	37	0,9982	0,9867	0,9339	0,7771	0,4890	0,1861	0,0301	0,0010	0,0000	0,0000	1,0000	1,0000	0,8965	0,0627
	38	0,9994	0,9943	0,9658	0,8610	0,6184	0,2893	0,0628	0,0032	0,0000	0,0000	1,0000	1,0000	0,9430	0,1173
	39	0,9998	0,9978	0,9840	0,9211	0,7378	0,4164	0,1199	0,0094	0,0000	0,0000	1,0000	1,0000	0,9716	0,2014
	40	0,9999	0,9992	0,9933	0,9598	0,8363	0,5563	0,2089	0,0245	0,0002	0,0000	1,0000	1,0000	0,9873	0,3170
	41	1,0000	0,9998	0,9975	0,9817	0,9084	0,6927	0,3319	0,0579	0,0008	0,0000	1,0000	1,0000	0,9950	0,4579
	42	1,0000	0,9999	0,9992	0,9927	0,9547	0,8096	0,4812	0,1221	0,0032	0,0000	1,0000	1,0000	0,9983	0,6089
	43	1,0000	1,0000	0,9998	0,9975	0,9806	0,8966	0,6387	0,2298	0,0118	0,0000	1,0000	1,0000	0,9995	0,7494
	44	1,0000	1,0000	0,9999	0,9993	0,9930	0,9520	0,7806	0,3839	0,0378	0,0000	1,0000	1,0000	0,9999	0,8612
	45	1,0000	1,0000	1,0000	0,9998	0,9979	0,9815	0,8879	0,5688	0,1036	0,0001	1,0000	1,0000	1,0000	0,9357
	46	1,0000	1,0000	1,0000	1,0000	0,9995	0.9943	0,9540	0,7497	0,2396	0,0016	1,0000	1,0000	1,0000	0,9762
	47	1,0000	1,0000	1,0000	1,0000	0,9999	0,9987	0,9858	0,8883	0,4595	0,0138	1,0000	1,0000	1,0000	0,9934
	48	1,0000	1,0000	1,0000	1,0000	1,0000	0,9998	0,9971	0,9662	0,7206	0,0894	1,0000	1,0000	1,0000	0,9988
	49	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9997	0,9948	0,9231	0,3950	1,0000	1,0000	1,0000	0,9999
	50	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000

DE STANDAARD-NORMALE VERDELING

Verdelingsfunctie $\Phi(z) = P(Z \leq z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}t^2} dt$

z	0	1	2	3	4	5	6	7	8	9
−0,0	0,5000	0,4960	0,4920	0,4880	0,4840	0,4801	0,4761	0,4721	0,4681	0,4641
−0,1	0,4602	0,4562	0,4522	0,4483	0,4443	0,4404	0,4364	0,4325	0,4286	0,4247
−0,2	0,4207	0,4168	0,4129	0,4090	0,4052	0,4013	0,3974	0,3936	0,3897	0,3859
−0,3	0,3821	0,3783	0,3745	0,3707	0,3669	0,3632	0,3594	0,3557	0,3520	0,3483
−0,4	0,3446	0,3409	0,3372	0,3336	0,3300	0,3264	0,3228	0,3192	0,3156	0,3121
−0,5	0,3085	0,3050	0,3015	0,2981	0,2946	0,2912	0,2877	0,2843	0,2810	0,2776
−0,6	0,2743	0,2709	0,2676	0,2643	0,2611	0,2578	0,2546	0,2514	0,2483	0,2451
−0,7	0,2420	0,2389	0,2358	0,2327	0,2296	0,2266	0,2236	0,2206	0,2177	0,2148
−0,8	0,2119	0,2090	0,2061	0,2033	0,2005	0,1977	0,1949	0,1922	0,1894	0,1867
−0,9	0,1841	0,1814	0,1788	0,1762	0,1736	0,1711	0,1685	0,1660	0,1635	0,1611
−1,0	0,1587	0,1562	0,1539	0,1515	0,1492	0,1469	0,1446	0,1423	0,1401	0,1379
−1,1	0,1357	0,1335	0,1314	0,1292	0,1271	0,1251	0,1230	0,1210	0,1190	0,1170
−1,2	0,1151	0,1131	0,1112	0,1093	0,1075	0,1056	0,1038	0,1020	0,1003	0,0985
−1,3	0,0968	0,0951	0,0934	0,0918	0,0901	0,0885	0,0869	0,0853	0,0838	0,0823
−1,4	0,0808	0,0793	0,0778	0,0764	0,0749	0,0735	0,0721	0,0708	0,0694	0,0681
−1,5	0,0668	0,0655	0,0643	0,0630	0,0618	0,0606	0,0594	0,0582	0,0571	0,0559
−1,6	0,0548	0,0537	0,0526	0,0516	0,0505	0,0495	0,0485	0,0475	0,0465	0,0455
−1,7	0,0446	0,0436	0,0427	0,0418	0,0409	0,0401	0,0392	0,0384	0,0375	0,0367
−1,8	0,0359	0,0351	0,0344	0,0336	0,0329	0,0322	0,0314	0,0303	0,0301	0,0294
−1,9	0,0287	0,0281	0,0274	0,0268	0,0262	0,0256	0,0250	0,0244	0,0239	0,0233
−2,0	0,0228	0,0222	0,0217	0,0212	0,0207	0,0202	0,0197	0,0192	0,0188	0,0183
−2,1	0,0179	0,0174	0,0170	0,0166	0,0162	0,0158	0,0154	0,0150	0,0146	0,0143
−2,2	0,0139	0,0136	0,0132	0,0129	0,0125	0,0122	0,0119	0,0116	0,0113	0,0110
−2,3	0,0107	0,0104	0,0102	0,0099	0,0096	0,0094	0,0091	0,0089	0,0087	0,0084
−2,4	0,0082	0,0080	0,0078	0,0075	0,0073	0,0071	0,0069	0,0068	0,0066	0,0064
−2,5	0,0062	0,0060	0,0059	0,0057	0,0055	0,0054	0,0052	0,0051	0,0049	0,0048
−2,6	0,0047	0,0045	0,0044	0,0043	0,0041	0,0040	0,0039	0,0038	0,0037	0,0036
−2,7	0,0035	0,0034	0,0033	0,0032	0,0031	0,0030	0,0029	0,0028	0,0027	0,0026
−2,8	0,0026	0,0025	0,0024	0,0023	0,0023	0,0022	0,0021	0,0021	0,0020	0,0019
−2,9	0,0019	0,0018	0,0018	0,0017	0,0016	0,0016	0,0015	0,0015	0,0014	0,0014
−3,0	0,0013	0,0013	0,0013	0,0012	0,0012	0,0011	0,0011	0,0011	0,0010	0,0010
−3,1	0,0010	0,0009	0,0009	0,0009	0,0008	0,0008	0,0008	0,0008	0,0007	0,0007
−3,2	0,0007	0,0007	0,0006	0,0006	0,0006	0,0006	0,0006	0,0005	0,0005	0,0005
−3,3	0,0005	0,0005	0,0005	0,0004	0,0004	0,0004	0,0004	0,0004	0,0004	0,0003
−3,4	0,0003	0,0003	0,0003	0,0003	0,0003	0,0003	0,0003	0,0003	0,0003	0,0002
−3,5	0,0002	0,0002	0,0002	0,0002	0,0002	0,0002	0,0002	0,0002	0,0002	0,0002
−3,6	0,0002	0,0002	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001

DE STANDAARD-NORMALE VERDELING (vervolg)

z	0	1	2	3	4	5	6	7	8	9
0,0	0,5000	0,5040	0,5080	0,5120	0,5160	0,5199	0,5239	0,5279	0,5319	0,5359
0,1	0,5398	0,5438	0,5478	0,5517	0,5557	0,5596	0,5636	0,5675	0,5714	0,5753
0,2	0,5793	0,5832	0,5871	0,5910	0,5948	0,5987	0,6026	0,6064	0,6103	0,6141
0,3	0,6179	0,6217	0,6255	0,6293	0,6331	0,6368	0,6406	0,6443	0,6480	0,6517
0,4	0,6554	0,6591	0,6628	0,6664	0,6700	0,6736	0,6772	0,6808	0,6844	0,6879
0,5	0,6915	0,6950	0,6985	0,7019	0,7054	0,7088	0,7123	0,7157	0,7190	0,7224
0,6	0,7257	0,7291	0,7324	0,7357	0,7389	0,7422	0,7454	0,7486	0,7517	0,7549
0,7	0,7580	0,7611	0,7642	0,7673	0,7704	0,7734	0,7764	0,7794	0,7823	0,7852
0,8	0,7881	0,7910	0,7939	0,7967	0,7995	0,8023	0,8051	0,8078	0,8106	0,8133
0,9	0,8159	0,8186	0,8212	0,8238	0,8264	0,8289	0,8315	0,8340	0,8365	0,8389
1,0	0,8413	0,8438	0,8461	0,8485	0,8508	0,8531	0,8554	0,8577	0,8599	0,8621
1,1	0,8643	0,8665	0,8686	0,8708	0,8729	0,8749	0,8770	0,8790	0,8810	0,8830
1,2	0,8849	0,8869	0,8888	0,8907	0,8925	0,8944	0,8962	0,8980	0,8997	0,9015
1,3	0,9032	0,9049	0,9066	0,9082	0,9099	0,9115	0,9131	0,9147	0,9162	0,9177
1,4	0,9192	0,9207	0,9222	0,9236	0,9251	0,9265	0,9279	0,9292	0,9306	0,9319
1,5	0,9332	0,9345	0,9357	0,9370	0,9382	0,9394	0,9406	0,9418	0,9429	0,9441
1,6	0,9452	0,9463	0,9474	0,9484	0,9495	0,9505	0,9515	0,9525	0,9535	0,9545
1,7	0,9554	0,9564	0,9573	0,9582	0,9591	0,9599	0,9608	0,9616	0,9625	0,9633
1,8	0,9641	0,9649	0,9656	0,9664	0,9671	0,9678	0,9686	0,9693	0,9699	0,9706
1,9	0,9713	0,9719	0,9726	0,9732	0,9738	0,9744	0,9750	0,9756	0,9761	0,9767
2,0	0,9772	0,9778	0,9783	0,9788	0,9793	0,9798	0,9803	0,9808	0,9812	0,9817
2,1	0,9821	0,9826	0,9830	0,9834	0,9838	0,9842	0,9846	0,9850	0,9854	0,9857
2,2	0,9861	0,9864	0,9868	0,9871	0,9875	0,9878	0,9881	0,9884	0,9887	0,9890
2,3	0,9893	0,9896	0,9898	0,9901	0,9904	0,9906	0,9909	0,9911	0,9913	0,9916
2,4	0,9918	0,9920	0,9922	0,9925	0,9927	0,9929	0,9931	0,9932	0,9934	0,9936
2,5	0,9938	0,9940	0,9941	0,9943	0,9945	0,9946	0,9948	0,9949	0,9951	0,9952
2,6	0,9953	0,9955	0,9956	0,9957	0,9959	0,9960	0,9961	0,9962	0,9963	0,9964
2,7	0,9965	0,9966	0,9967	0,9968	0,9969	0,9970	0,9971	0,9972	0,9973	0,9974
2,8	0,9974	0,9975	0,9976	0,9977	0,9977	0,9978	0,9979	0,9979	0,9980	0,9981
2,9	0,9981	0,9982	0,9982	0,9983	0,9984	0,9984	0,9985	0,9985	0,9986	0,9986
3,0	0,9987	0,9987	0,9987	0,9988	0,9988	0,9989	0,9989	0,9989	0,9990	0,9990
3,1	0,9990	0,9991	0,9991	0,9991	0,9992	0,9992	0,9992	0,9992	0,9993	0,9993
3,2	0,9993	0,9993	0,9994	0,9994	0,9994	0,9994	0,9994	0,9995	0,9995	0,9995
3,3	0,9995	0,9995	0,9995	0,9996	0,9996	0,9996	0,9996	0,9996	0,9996	0,9997
3,4	0,9997	0,9997	0,9997	0,9997	0,9997	0,9997	0,9997	0,9997	0,9997	0,9998
3,5	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998
3,6	0,9998	0,9998	0,9999	0,9999	0,9999	0,9999	0,9999	0,9999	0,9999	0,9999

Overschrijdingskansen (één-, resp. tweezijdig)

z	0,675	1,282	1,645	1,960	2,326	2,576	3,090
$P(Z > z)$	0,25	0,10	0,05	0,025	0,01	0,005	0,001
$P(\|Z\| > z)$	0.50	0,20	0,10	0,05	0,02	0,01	0,002