CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam "Wiskunde A"

Date: 16 January 2015
Time: 13.30 – 16.30
Questions: 7

Please read the instructions below carefully before answering the questions.

Make sure your name is clearly written on each answer sheet.

Take a new answer sheet for each question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your graphing calculator are invalid.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use Tipp-ex.

You can use a (graphing) calculator. The use of hand-held computers is not allowed. If there is doubt about the status of your equipment, the invigilator will decide whether it is allowed for use during the exam.

On the last pages of this exam you will find a list of formulas that you may use during this exam. Tables of the binomial and normal probability distributions are available from the invigilator. The use of other formula sheets or books (like BINAS) is not allowed.

You can use a dictionary if it is approved by the invigilator.

During the exam, the use of a mobile telephone or other electrical equipment is not allowed. Please switch off your mobile telephone.

Please check www.ccvx.nl for more information on this exam (unfortunately most information is available in Dutch only). Answers to the questions of this exam will be published on this website early next week.

Points that can be scored for each sub-question:

Question	1	2	3	4	5	6	7
a	4	3	5	4	2	6	4
b	4	3	6	5	2	6	4
c	4	2			4	4	5
d		3					4
e		1					
f		5					
Total	12	17	11	9	8	16	17

$$\text{Grade} = \frac{\text{total points scored}}{10} + 1$$

You will pass the exam if your grade is at least 5.5

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Question 1 – Three functions

Given the function $f(x) = x^3 - 8x^2 + 5x$.

4pt   a   Compute algebraically the x-coordinates of the points on the graph of f where the tangent to this graph is horizontal.

A is the point on the graph of f for which $x_A = 1$. B is the point on the graph of f for which $x_B = 6$. The graph of the first degree function g passes through points A and B.

4pt   b   Find a formula for g.

The graph of the exponential function h passes through the points (2,9) and (4,16).

4pt   c   Find a formula for h.

Question 2 – Noflu

The pharmaceutical company Pillfit produces a new medicine with the name Noflu.
The company claims that 10% of the patients who take this medicine will be healed from influenza after one day. *In sub-questions a, b, c and d you may assume that this claim is true. Give your answers to sub-questions a and b rounded off to four decimals.*

20 influenza patients take this new medicine.

3pt   a   Compute the probability that none of these 20 patients is healed from influenza after one day.

3pt   b   Compute the probability that at least three of these 20 patients are healed from influenza after one day.

Pillfit wants to observe a group of randomly selected patients to study the effects of Noflu. In this experiment, the probability that at least one of the patients will be healed after one day, should be over 95%.

2pt   c   Show that for a group of n patients, this probability is given by $1 - 0.9^n$.

3pt   d   How large must the group be at least in order to obtain that the probability that at least one patient in the group heals in one day is over 95%?

Pillfit also claims that 50% of patients who use Noflu will be healed from influenza after two days. The Health counsel suspects that this percentage will be lower and decides to test this.

1pt   e   State the null hypothesis and the alternative hypothesis for this test.

Fifty randomly selected patients are given Noflu. After two days, 32 of these patients still have influenza.

5pt   f   Will the null hypothesis be rejected if the level of significance is $\alpha = 0{,}05$?

Question 3 – Missing pills

A new employee accidently puts two pills of Noflu in a box in which there already are three peppermints. These peppermints are very similar to the Noflu pills.
The employee wants to correct his mistake as soon as possible. Therefore, he randomly picks objects (pills or peppermints) from the box until he has recovered the two Noflu pills.
The number of objects that he picks from the box until he has recovered the two Noflu pills is a random variable X.

5pt   a   Show that $P(X = 3) = 0.2$ and $P(X = 4) = 0.3$

6pt   b   Compute $E(X)$

Question 4 – Headache

Pillfit also produces headache pills. The weights of these pills are normally distributed with an average of 2.0 grams and a standard deviation of 0.1 grams.
These pills are sold in boxes of 50. The weights of these boxes are also normally distributed. The average weight of the boxes is 10 grams and the standard deviation is 0.5 grams. The 50 pills in a box are randomly selected.
Give your answers to sub-questions a and b rounded off to four decimals.

4pt   a   Compute the probability that the average weight of 50 randomly selected pills is under 1.95 grams.

5pt   b   Compute the probability that the total weight of a box of 50 pills (including the weight of the box) is over 111 grams.

Question 5 – Up and down

A metal sphere is hung from a ceiling by means of a spring. At a certain moment, the sphere is pulled down and then released. From this moment onwards the sphere swings up and down. The distance between the center of the sphere and the ceiling is given by the formula

$$y(t) = 12.5 + 4.5 \cos\left(\tfrac{1}{3}\pi t\right)$$

In this formula, t is the time in seconds and y is the distance between the center of the sphere and the ceiling in centimeters.

2pt   a   Compute the minimal and the maximal distance between the center of the sphere and the ceiling.

2pt   b   Compute the distance between the center of the sphere and the ceiling at $t = 1$.

4pt   c   Use the period of $y(t)$ to determine the first three times after $t = 1$ at which the bullet is on the same height as it is on $t = 1$.

Question 6 – Butcher Baker

Butcher Baker sells first quality meat at the Market Square. His brother, a well-known economist, has analyzed the sales figures of Butcher Baker and he has derived two models for the relation between the profit Butcher Baker makes on a day and the quantity of meat that he buys in for that day.

In the first model, this relation is given by

$$W = 18 - 3Q - \frac{64}{3Q + 2}$$

In this formula, W is the profit in hundreds of euros and Q is the quantity of meat bought in for a day in hundreds of kilograms.

In the figure to the right a sketch of the graph of this function is shown.
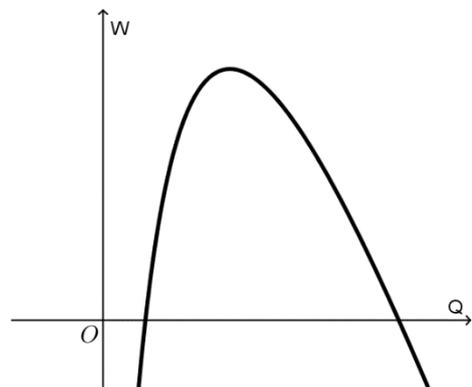
6pt   a   Compute algebraically the values of Q for which $W = 0$.
           What is the economical meaning of these values?

6pt   a   Compute algebraically the maximum profit that Butcher Baker can make in a day.

In the second model, the relation between W and Q is given by

$$^2\log Q = 2 \cdot \,^2\log(W + 4) - 5$$

4pt   c   Transform this formula algebraically to a formula of the form $W = a \cdot Q^n + b$.

Question 7 - Tsunami

A tsunami is a tidal wave caused by a submarine earthquake. The speed at which a tsunami moves can be approximated by the formula

$$v = 34\sqrt{\frac{d}{9}}$$

In this formula, v is the speed in km/hour and d is the depth of the water in meters.

4pt a Compute the speed of the tsunami in meters per second given that the depth of the water is 100 meter.

4pt b Compute algebraically the depth of the water given that the speed of the tsunami is 170 km/hour.

There is also a relation between the height h of the tidal wave and the depth of the water d. This relation is given by the formula

$$h = \frac{4}{3 \cdot \sqrt[4]{d}}$$

with h and d in meters.

The derivative of h as a function of d gives the rate of change (in meter height per meter depth) of the height of the tidal wave if the depth of the water changes.

5pt c Calculate this change algebraically given that the depth of the water is 16 meter.

In a certain coastal region, the depth of the water is given by the formula

$$d = 81x^2$$

with d the depth of the water in meters and x the distance to the coast in km.

4pt d Find a formula for h, the height of the tidal wave in meters, as a function of x, the distance to the coast in km. Simplify this formula as far as possible.

List of formulas for the exam wiskunde A

Probability

If X and Y are any random variables, then: $E(X + Y) = E(X) + E(Y)$

If furthermore X and Y are independent, then: $\sigma(X + Y) = \sqrt{\sigma^2(X) + \sigma^2(Y)}$

$\sqrt{n}$-law:

For n independent repetitions of the same experiment where the result of each experiment is a random variable X, the sum of the results is a random variable S and the mean of the results is a random variable $\overline{X}$.

$$E(S) = n \cdot E(X) \qquad\qquad \sigma(S) = \sqrt{n} \cdot \sigma(X)$$

$$E(\overline{X}) = E(X) \qquad\qquad \sigma(\overline{X}) = \frac{\sigma(X)}{\sqrt{n}}$$

Binomial Distribution

If X has a binomial distribution with parameters n (number of experiments) and p (probability of success at each experiment), then

$$P(X = k) = \binom{n}{k} \cdot p^k \cdot (1 - p)^{n-k} \quad \text{with} \quad k = 0, 1, 2, \ldots, n$$

Expectation: $E(X) = np$ \qquad Standard deviation: $\sigma(X) = \sqrt{n \cdot p \cdot (1 - p)}$

Normal Distribution

If X is a normally distributed random variable with mean μ and standard deviation σ, then

$Z = \dfrac{X - \mu}{\sigma}$ has a standard normal distribution and $P(X < g) = P\left(Z < \dfrac{g - \mu}{\sigma}\right)$

Differentiation

rule	function	derivative function
Sum rule	$s(x) = f(x) + g(x)$	$s'(x) = f'(x) + g'(x)$
Product rule	$p(x) = f(x) \cdot g(x)$	$p'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$
Quotient rule	$q(x) = \dfrac{f(x)}{g(x)}$	$q'(x) = \dfrac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{\big(g(x)\big)^2}$
Chain rule	$k(x) = f(g(x))$	$k'(x) = f'\big(g(x)\big) \cdot g'(x)$ or $\dfrac{dk}{dx} = \dfrac{df}{dg} \cdot \dfrac{dg}{dx}$

More formulas on the next page.

List of formulas for the exam wiskunde A (continued)

Logarithms

rule	conditions
${}^g\!\log a + {}^g\!\log b = {}^g\!\log ab$	$g > 0,\ \ g \neq 1,\ \ a > 0,\ \ b > 0$
${}^g\!\log a - {}^g\!\log b = {}^g\!\log \dfrac{a}{b}$	$g > 0,\ \ g \neq 1,\ \ a > 0,\ \ b > 0$
${}^g\!\log a^p = p \cdot {}^g\!\log a$	$g > 0,\ \ g \neq 1,\ \ a > 0$
${}^g\!\log a = \dfrac{{}^p\!\log a}{{}^p\!\log g}$	$g > 0,\ \ g \neq 1,\ \ a > 0,\ \ p > 0,\ \ p \neq 1$

Arithmetic and geometric sequences

Arithmetic sequence:	$Sum = \frac{1}{2} \cdot number\ of\ terms \cdot (u_e + u_l)$
Arithmetic sequence:	$Sum = \dfrac{u_{l+1} - u_e}{r - 1} \quad (r \neq 1)$
In both formulas:	e =number first term of the sum; l = number last term of the sum

End of the exam.

Is your name on all answer sheets?

Tables of the binomial and normal probability distributions are available from the invigilator.

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