CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam "Wiskunde A"

Date: 28 July 2014

Time: 14.00 – 17.00

Please read the instructions below carefully before answering the questions.

- This exam consists of 7 questions, with in total 23 sub-questions.
- Points that can be scored:

question	1	2	3	4	5	6	7
a	4	3	2	5	3	4	4
b	4	7	3	4	3	4	4
c	4		2	4	4	5	2
d	5		4				6
total	17	10	11	13	10	13	16

  You will pass the exam if you score a total of at least 45 points out of a possible 90 points.
- Make sure your name is clearly written on every answer sheet.
- Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your (graphing) calculator are invalid.
- Write your answers in ink. Do not use a pencil, except when drawing graphs.
- You can use a (graphing) calculator. The use of hand-held computers is not allowed. If there is doubt about the status of your equipment, the exam monitor will decide whether it is allowed for use during the exam.
- On page 6 you will find a list of formulas that you may use during this exam. On the last pages of this exam you will find tables of the binomial and standard normal distributions. The use of other formula sheets or books (like BINAS) is not allowed.
- You can use a dictionary if it is approved by the exam monitor.
- Please switch off your mobile telephone.
- Please check www.ccvx.nl for further information on this exam (unfortunately in Dutch only). Answers to the questions will be published on this website next week. Do not call the Open Universiteit, since they do not have any further information about this exam.

1 Functions

Given is a function f with $f(1) = 256$ and $f(5) = 2401$.

4 pt **a** Compute $f(2)$ in case this is a linear function.

4 pt **b** Also compute $f(2)$ in case this is an exponential function.

The function g is given by $g(x) = 2x^3 - 15x^2 + 36x$.

4 pt **c** Solve algebraically: $g(x) = 0$.

There are two points on the graph of g where the function has an extreme value.

5 pt **d** Compute the coordinates of these two points algebraically.

2 Garden chairs

A factory produces luxury garden chairs.
The selling price in euros (p) of these chairs is a function of q, the number of chairs that are produced. This function is given by the formula

$$p(q) = 250 - 4\sqrt{q}$$

The cost in euros (C) for the production of these chairs is given by

$$C(q) = 50{,}000 + 40q$$

3 pt **a** Show that the profit in euros (P) is given by

$$P(q) = 210q - 4q\sqrt{q} - 50{,}000$$

7 pt **b** Compute the maximal profit algebraically and compute the price of the garden chairs when the profit is maximal.

3 Ferris wheel

Jack takes a ride on a Ferris wheel (the big wheel on a country fair). First, the gondolas one by one stop at the entry platform which is located at the lowest point of the Ferris wheel. Once the gondolas are filled, the Ferris wheel runs for five minutes. The height (in meters above street level) of Jack's gondola is called H. During the five minutes that the Ferris wheel is turning, this height is given by the formula

$$H(t) = 11 + 10\sin\left(\pi(\tfrac{8}{5}t + \tfrac{5}{6})\right) \quad (t \text{ in minutes})$$

2 pt **a** At what height is the floor of Jack's gondola at $t = 0$?

3 pt **b** Is Jack's gondola moving upwards or downwards at $t = 0$? Explain!

2 pt **c** Compute algebraically the height of the floor of a gondola at the entry platform.

4 pt **d** Compute algebraically the number of rounds that the Ferris wheel turns in five minutes.

4 Waste

In Wasteland, currently each week 50 kilotons of waste is given to the garbage man. There is a campaign to reduce the amount of waste. According to research by the agency that runs this campaign, the amount of waste will be reduced according to the formula

$$A = \frac{1400t + 7000}{98t + 140}$$

In this formula, A is the amount of waste in kilotons per week and t is the time in years, with $t = 0$ on 1 August 2014.

5 pt **a** Use the derivative function $\dfrac{\mathrm{d}A}{\mathrm{d}t}$ to show that the amount of waste is indeed decreasing according to this formula.

4 pt **b** Compute algebraically in which year the amount of waste will be reduced to 20 kilotons according to this formula.

According to an environmental movement, the weekly amount of waste is not given by the formula of the campaign agency, but by the formula

$$A = 50 - 15\log(9t + 1)$$

with A and t as indicated above.

4 pt **c** Compute algebraically in which year the amount of waste is reduced to 20 kilotons per week according to the formula of the environmental movement.

5 Internet in the train?

In many Dutch intercity trains you can use free internet. On the line between Zwolle and Arnhem, 60% of the trains is equipped with an internet connection. However, if there is such a connection in the train, there is a 25% chance that a passenger is not able to connect to the internet.
Jackie often travels by train between Zwolle and Arnhem. She does not travel at a fixed time, so we can assume that the availability of the internet for all these train rides are independent events.

On a day Jackie needs to send an e-mail, but she has only time to do so during the train ride. She travels from Zwolle to Arnhem in the morning of that day and she goes back in the afternoon. If she cannot send her e-mail on the morning ride, she will try again in the afternoon.

3 pt **a** Show that the probability that Jackie cannot send her e-mail on the morning ride is equal to 0.55.

3 pt **b** Compute the probability that she sends the e-mail on the afternoon ride.

4 pt **c** Compute the probability that on the next 20 train rides that Jackie makes between Zwolle and Arnhem, she wil not be able to send an e-mail in more than 9 cases.
Give your answer rounded off to four digits behind the decimal dot.

6 Prizes and vouchers

Shopping Center "The Garden" celebrates its 25th anniversary with a wheel of fortune. For every 25 euro you spend at the shopping center, you can give a twist to the wheel of fortune. This wheel consists of 12 equally sized boxes, which at every new twist have the same probability of being drawn. On each box, the prize that you can win is indicated. On the wheel, there are 8 prizes with a value of 5 euro, 3 prizes with a value of 10 euro and there is one prize with a value of 25 euro. At the beginning of the celebration a large number of all prizes is available.

On the first day of this celebration, Annie spends 75 euro at the shopping center, so she can twist the wheel of fortune three times and she gets three prizes.

4 pt **a** Compute the probability that the total value of these prizes equals 20 euro.

4 pt **b** Compute the probability that the total value of these prizes is larger than 30 euro.
Give your answer rounded off to four digits behind the decimal dot.

On the last day of this celebration, almost all available prizes are given away. Therefore, a box is made which contains 120 enveloppes. In 80 of these enveloppes there is a voucher of 10 euros, in 30 of these enveloppes there is a vouchers of 20 euros and in the remaining 10 enveloppes there is a voucher of 30 euros. The enveloppes are well mixed and from the outside the value of the coupon cannot be determined. On this last day, the first 120 customers may randomly select one enveloppe from this box.

5 pt **c** Compute the probability that the total value of the coupons that the first three customers of this day get, equals 50 euro.
Give your answer rounded off to four digits behind the decimal dot.

7 Farmer Fred

Last year, the weight of the apples from the harvest of Farmer Fred was normally distributed with an average of 99 grammes an a standard deviation of 10 grammes.
In a box, there are 25 randomly selected apples from this harvest.

4 pt **a** Compute the probability that the total weight of these 25 apples is larger than 2500 grammes.
Give your answer rounded off to four digits behind the decimal dot.

The apples are divided into three weight classes:
Small: up to 90 grammes
Middle: from 90 grammes to 110 grammes
Big: over 110 grammes

4 pt **b** How many of the 25 apples in the box can be expected in the Middle class?

Farmer Fred wants to know whether the average weight of the apples in this year's harvest is equal to last year's 99 grammes. To test this, he weighs 9 randomly selected apples from this year's harvest. In this test, he assumes that the weight is again normally distributed with a standard deviation of 10 grammes and he takes significance level $\alpha = 0.05$.

2 pt **c** State the null hypothesis and the alternative hypotheses for this test.

The weights of these 9 apples are: 89, 101, 90, 93, 96, 85, 91, 92 and 100 grammes.

6 pt **d** What is the conclusion of this test?

List of formulas for the exam Wiskunde A

Probability

If X and Y are any random variables, then: $\mathrm{E}(X+Y) = \mathrm{E}(X) + \mathrm{E}(Y)$
If furthermore X and Y are independent, then: $\sigma(X+Y) = \sqrt{\sigma^2(X) + \sigma^2(Y)}$

$\sqrt{n}$-law:
For n independent repetitions of the same experiment where the result of each experiment is a random variable X, the sum of the results is a random variable S and the mean of the results is a random variable $\overline{X}$.

$\mathrm{E}(S) = n \cdot \mathrm{E}(X)$ $\qquad$ $\sigma(S) = \sqrt{n} \cdot \sigma(X)$

$\mathrm{E}(\overline{X}) = \mathrm{E}(X)$ $\qquad$ $\sigma(\overline{X}) = \dfrac{\sigma(X)}{\sqrt{n}}$

Binomial Distribution

If X has a binomial distribution with parameters n (number of experiments) and p (probability of succes at each experiment), then:

$\mathrm{P}(X = k) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}$ with $k = 0, 1, 2, \ldots, n$

Expectation: $\mathrm{E}(X) = n \cdot p$ $\qquad$ Standard deviation: $\sigma(X) = \sqrt{n \cdot p \cdot (1-p)}$

Normal Distribution

If X is a normally distributed random variable with mean μ and standard deviation σ, then:

$Z = \dfrac{X-\mu}{\sigma}$ has the standard normal distribution and $\mathrm{P}(X < g) = \mathrm{P}\left(Z < \dfrac{g-\mu}{\sigma}\right)$

Differentiation

rule	**function**	**derivative function**
Sum rule	$s(x) = f(x) + g(x)$	$s'(x) = f'(x) + g'(x)$
Product rule	$p(x) = f(x) \cdot g(x)$	$p'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$
Quotient rule	$q(x) = \dfrac{f(x)}{g(x)}$	$q'(x) = \dfrac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$
Chain rule	$k(x) = f(g(x))$	$k'(x) = f'(g(x)) \cdot g'(x)$ or $\dfrac{\mathrm{d}k}{\mathrm{d}x} = \dfrac{\mathrm{d}f}{\mathrm{d}g} \cdot \dfrac{\mathrm{d}g}{\mathrm{d}x}$

Logarithms

rule	**conditions**
$\log_g a + \log_g b = \log_g ab$	$g > 0,\ g \neq 1,\ a > 0,\ b > 0$
$\log_g a - \log_g b = \log_g \dfrac{a}{b}$	$g > 0,\ g \neq 1,\ a > 0,\ b > 0$
$\log_g a^p = p \cdot \log_g a$	$g > 0,\ g \neq 1,\ a > 0$
$\log_g a = \dfrac{\log_p a}{\log_p g}$	$g > 0,\ g \neq 1,\ a > 0,\ p > 0,\ p \neq 1$

Arithmetic and geometric sequences

Arithmetic sequence:	Sum $= \frac{1}{2} \cdot$ number of terms $\cdot (u_e + u_l)$
Geometric sequence:	Sum $= \dfrac{u_{l+1} - u_e}{r-1}$ $(r \neq 1)$
In both formulas	e = number first term of the sum; l = number last term of the sum.

CUMULATIEVE BINOMIALE VERDELINGEN (vervolg)

Verdelingsfunctie $B_{n,p}(x) = P(X \leqq x) = \sum_{k=0}^{k=x} \binom{n}{k} p^k (1-p)^{n-k}$

		p									
n	*x*	0,0500	0,1000	0,1500	0,2000	0,2500	0,3000	0,3500	0,4000	0,4500	0,5000
10	0	0,5987	0,3487	0,1969	0,1074	0,0563	0,0282	0,0135	0,0060	0,0025	0,0010
	1	0,9139	0,7361	0,5443	0,3758	0,2440	0,1493	0,0860	0,0464	0,0233	0,0107
	2	0,9885	0,9298	0,8202	0,6778	0,5256	0,3828	0,2616	0,1673	0,0996	0,0547
	3	0,9990	0,9872	0,9500	0,8791	0,7759	0,6496	0,5138	0,3823	0,2660	0,1719
	4	0,9999	0,9984	0,9901	0,9672	0,9219	0,8497	0,7515	0,6331	0,5044	0,3770
	5	1,0000	0,9999	0,9986	0,9936	0,9803	0,9527	0,9051	0,8338	0,7384	0,6230
	6	1,0000	1,0000	0,9999	0,9991	0,9965	0,9894	0,9740	0,9452	0,8980	0,8281
	7	1,0000	1,0000	1,0000	0,9999	0,9996	0,9984	0,9952	0,9877	0,9726	0,9453
	8	1,0000	1,0000	1,0000	1,0000	1,0000	0,9999	0,9995	0,9983	0,9955	0,9893
	9	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9999	0,9997	0,9990
	10	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000

CUMULATIEVE BINOMIALE VERDELINGEN (vervolg)

Verdelingsfunctie $B_{n,p}(x) = P(X \leqq x) = \sum_{k=0}^{k=x} \binom{n}{k} p^k (1-p)^{n-k}$

		p									
n	*x*	0,0500	0,1000	0,1500	0,2000	0,2500	0,3000	0,3500	0,4000	0,4500	0,5000
20	0	0,3585	0,1216	0,0388	0,0115	0,0032	0,0008	0,0002	0,0000	0,0000	0,0000
	1	0,7358	0,3917	0,1756	0,0692	0,0243	0,0076	0,0021	0,0005	0,0001	0,0000
	2	0,9245	0,6769	0,4049	0,2061	0,0913	0,0355	0,0121	0,0036	0,0009	0,0002
	3	0,9841	0,8670	0,6477	0,4114	0,2252	0,1071	0,0444	0,0160	0,0049	0,0013
	4	0,9974	0,9568	0,8298	0,6296	0,4148	0,2375	0,1182	0,0510	0,0189	0,0059
	5	0,9997	0,9887	0,9327	0,8042	0,6172	0,4164	0,2454	0,1256	0,0553	0,0207
	6	1,0000	0,9976	0,9781	0,9133	0,7858	0,6080	0,4166	0,2500	0,1299	0,0577
	7	1,0000	0,9996	0,9941	0,9679	0,8982	0,7723	0,6010	0,4159	0,2520	0,1316
	8	1,0000	0,9999	0,9987	0,9900	0,9591	0,8867	0,7624	0,5956	0,4143	0,2517
	9	1,0000	1,0000	0,9998	0,9974	0,9861	0,9520	0,8782	0,7553	0,5914	0,4119
	10	1,0000	1,0000	1,0000	0,9994	0,9961	0,9829	0,9468	0,8725	0,7507	0,5881
	11	1,0000	1,0000	1,0000	0,9999	0,9991	0,9949	0,9804	0,9435	0,8692	0,7483
	12	1,0000	1,0000	1,0000	1,0000	0,9998	0,9987	0,9940	0,9790	0,9420	0,8684
	13	1,0000	1,0000	1,0000	1,0000	1,0000	0,9997	0,9985	0,9935	0,9786	0,9423
	14	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9997	0,9984	0,9936	0,9793
	15	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9997	0,9985	0,9941
	16	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9997	0,9987
	17	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9998
	18	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	19	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	20	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000

CUMULATIEVE BINOMIALE VERDELINGEN (vervolg)

	p												
0,5500	0,6000	0,6500	0,7000	0,7500	0,8000	0,8500	0,9000	0,9500	1/8	1/3	2/3	5/6
0,0003	0,0001	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,1615	0,0173	0,0000	0,0000
0,0045	0,0017	0,0005	0,0001	0,0000	0,0000	0,0000	0,0000	0,0000	0,4845	0,1040	0,0004	0,0000
0,0274	0,0123	0,0048	0,0016	0,0004	0,0001	0,0000	0,0000	0,0000	0,7752	0,2991	0,0034	0,0000
0,1020	0,0548	0,0260	0,0106	0,0035	0,0009	0,0001	0,0000	0,0000	0,9303	0,5593	0,0197	0,0003
0,2616	0,1662	0,0949	0,0473	0,0197	0,0064	0,0014	0,0001	0,0000	0,9845	0,7869	0,0766	0,0024
0,4956	0,3669	0,2485	0,1503	0,0781	0,0328	0,0099	0,0016	0,0001	0,9976	0,9234	0,2131	0,0155
0,7340	0,6177	0,4862	0,3504	0,2241	0,1209	0,0500	0,0128	0,0010	0,9997	0,9803	0,4407	0,0697
0,9004	0,8327	0,7384	0,6172	0,4744	0,3222	0,1798	0,0702	0,0115	1,0000	0,9966	0,7009	0,2248
0,9767	0,9536	0,9140	0,8507	0,7560	0,6242	0,4557	0,2639	0,0861	1,0000	0,9996	0,8960	0,5155
0,9975	0,9940	0,9865	0,9718	0,9437	0,8926	0,8031	0,6513	0,4013	1,0000	1,0000	0,9827	0,8385
1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000

CUMULATIEVE BINOMIALE VERDELINGEN (vervolg)

	p												
0,5500	0,6000	0,6500	0,7000	0,7500	0,8000	0,8500	0,9000	0,9500	1/8	1/3	2/3	5/6
0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0261	0,0003	0,0000	0,0000
0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,1304	0,0033	0,0000	0,0000
0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,3287	0,0176	0,0000	0,0000
0,0003	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,5665	0,0604	0,0000	0,0000
0,0015	0,0003	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,7687	0,1515	0,0000	0,0000
0,0064	0,0016	0,0003	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,8982	0,2972	0,0002	0,0000
0,0214	0,0065	0,0015	0,0003	0,0000	0,0000	0,0000	0,0000	0,0000	0,9629	0,4793	0,0009	0,0000
0,0580	0,0210	0,0060	0,0013	0,0002	0,0000	0,0000	0,0000	0,0000	0,9887	0,6615	0,0037	0,0000
0,1308	0,0565	0,0196	0,0051	0,0009	0,0001	0,0000	0,0000	0,0000	0,9972	0,8095	0,0130	0,0000
0,2493	0,1275	0,0532	0,0171	0,0039	0,0006	0,0000	0,0000	0,0000	0,9994	0,9081	0,0376	0,0001
0,4086	0,2447	0,1218	0,0480	0,0139	0,0026	0,0002	0,0000	0,0000	0,9999	0,9624	0,0919	0,0006
0,5857	0,4044	0,2376	0,1133	0,0409	0,0100	0,0013	0,0001	0,0000	1,0000	0,9870	0,1905	0,0028
0,7480	0,5841	0,3990	0,2277	0,1018	0,0321	0,0059	0,0004	0,0000	1,0000	0,9963	0,3385	0,0113
0,8701	0,7500	0,5834	0,3920	0,2142	0,0867	0,0219	0,0024	0,0000	1,0000	0,9991	0,5207	0,0371
0,9447	0,8744	0,7546	0,5836	0,3828	0,1958	0,0673	0,0113	0,0003	1,0000	0,9998	0,7028	0,1018
0,9811	0,9490	0,8818	0,7625	0,5852	0,3704	0,1702	0,0432	0,0026	1,0000	1,0000	0,8485	0,2313
0,9951	0,9840	0,9556	0,8929	0,7748	0,5886	0,3523	0,1330	0,0159	1,0000	1,0000	0,9396	0,4335
0,9991	0,9964	0,9879	0,9645	0,9087	0,7939	0,5951	0,3231	0,0755	1,0000	1,0000	0,9824	0,6713
0,9999	0,9995	0,9979	0,9924	0,9757	0,9308	0,8244	0,6083	0,2642	1,0000	1,0000	0,9967	0,8696
1,0000	1,0000	0,9998	0,9992	0,9968	0,9885	0,9612	0,8784	0,6415	1,0000	1,0000	0,9997	0,9739
1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000

CUMULATIEVE BINOMIALE VERDELINGEN (vervolg)

Verdelingsfunctie $B_{n,p}(x) = P(X \leqq x) = \sum_{k=0}^{k=x} \binom{n}{k} p^k (1-p)^{n-k}$

		p										
n	x	0,0100	0,0500	0,1000	0,1500	0,2000	0,2500	0,3000	0,3500	0,4000	0,4500	0,5000
50	0	0,6050	0,0769	0,0052	0,0003	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
	1	0,9106	0,2794	0,0338	0,0029	0,0002	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
	2	0,9862	0,5405	0,1117	0,0142	0,0013	0,0001	0,0000	0,0000	0,0000	0,0000	0,0000
	3	0,9984	0,7604	0,2503	0,0460	0,0057	0,0005	0,0000	0,0000	0,0000	0,0000	0,0000
	4	0,9999	0,8964	0,4312	0,1121	0,0185	0,0021	0,0002	0,0000	0,0000	0,0000	0,0000
	5	1,0000	0,9622	0,6161	0,2194	0,0480	0,0070	0,0007	0,0001	0,0000	0,0000	0,0000
	6	1,0000	0,9882	0,7702	0,3613	0,1034	0,0194	0,0025	0,0002	0,0000	0,0000	0,0000
	7	1,0000	0,9968	0,8779	0,5188	0 1904	0,0453	0,0073	0,0008	0,0001	0,0000	0,0000
	8	1,0000	0,9992	0,9421	0,6681	0,3073	0,0916	0,0183	0 0025	0,0002	0,0000	0,0000
	9	1,0000	0,9998	0,9755	0,7911	0,4437	0,1637	0,0402	0,0067	0,0008	0,0001	0,0000
	10	1,0000	1,0000	0,9906	0,8801	0,5836	0,2622	0,0789	0,0160	0,0022	0,0002	0,0000
	11	1,0000	1,0000	0,9968	0,9372	0,7107	0,3816	0,1390	0,0342	0,0057	0,0006	0,0000
	12	1,0000	1,0000	0,9990	0,9699	0,8139	0,5110	0,2229	0,0661	0,0133	0,0018	0,0002
	13	1,0000	1,0000	0,9997	0,9868	0,8894	0,6370	0,3279	0,1163	0,0280	0,0045	0,0005
	14	1,0000	1,0000	0,9999	0,9947	0,9393	0,7481	0,4468	0,1878	0,0540	0,0104	0,0013
	15	1,0000	1,0000	1,0000	0,9981	0,9692	0,8369	0,5692	0,2801	0,0955	0,0220	0,0033
	16	1,0000	1,0000	1,0000	0,9993	0,9856	0,9017	0,6839	0,3889	0,1561	0,0427	0,0077
	17	1,0000	1,0000	1,0000	0,9998	0,9937	0,9449	0,7822	0,5060	0,2369	0,0765	0,0164
	18	1,0000	1,0000	1,0000	0,9999	0,9975	0,9713	0,8594	0,6216	0,3356	0,1273	0,0325
	19	1,0000	1,0000	1,0000	1,0000	0,9991	0,9861	0,9152	0,7264	0,4465	0,1974	0,0595
	20	1,0000	1,0000	1,0000	1,0000	0,9997	0,9937	0,9522	0,8139	0,5610	0,2862	0,1013
	21	1,0000	1,0000	1,0000	1,0000	0,9999	0,9974	0,9749	0,8813	0,6701	0,3900	0,1611
	22	1,0000	1,0000	1,0000	1,0000	1,0000	0,9990	0,9877	0,9290	0,7660	0,5019	0,2399
	23	1,0000	1,0000	1,0000	1,0000	1,0000	0,9996	0,9944	0,9604	0,8438	0,6134	0,3359
	24	1,0000	1,0000	1,0000	1,0000	1,0000	0,9999	0,9976	0,9793	0,9022	0,7160	0,4439
	25	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9991	0,9900	0,9427	0,8034	0,5561
	26	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9997	0,9955	0,9686	0,8721	0,6641
	27	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9999	0,9981	0,9840	0,9220	0,7601
	28	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9993	0,9924	0,9556	0,8389
	29	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9997	0,9966	0,9765	0,8987
	30	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9999	0,9986	0,9884	0,9405
	31	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9995	0,9947	0,9675
	32	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9998	0,9978	0,9836
	33	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9999	0,9991	0,9923
	34	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9997	0,9967
	35	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9999	0,9987
	36	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9995
	37	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9998
	38	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	39	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	40	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	41	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	42	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	43	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	44	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	45	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	46	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	47	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	48	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	49	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	50	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000

CUMULATIEVE BINOMIALE VERDELINGEN (vervolg)

p														
0,5500	0,6000	0,6500	0,7000	0,7500	0,8000	0,8500	0,9000	0,9500	0,9900	[illegible]	[illegible]	[illegible]	[illegible]
0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0000
0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0012	0,0000	0,0000	0,0000
0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0066	0,0000	0,0000	0,0000
0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0238	0,0000	0,0000	0,0000
0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0643	0,0000	0,0000	0,0000
0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,1388	0,0001	0,0000	0,0000
0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,2506	0,0005	0,0000	0,0000
0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,3911	0,0017	0,0000	0,0000
0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,5421	0,0050	0,0000	0,0000
0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,6830	0,0127	0,0000	0,0000
0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,7986	0,0284	0,0000	0,0000
0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,8827	0,0570	0,0000	0,0000
0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,9373	0,1035	0,0000	0,0000
0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,9693	0,1715	0,0000	0,0000
0,0001	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,9862	0,2612	0,0000	0,0000
0,0003	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,9943	0,3690	0,0000	0,0000
0,0009	0,0001	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,9978	0,4868	0,0000	0,0000
0,0022	0,0002	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,9992	0,6046	0,0000	0,0000
0,0053	0,0005	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,9997	0,7126	0,0000	0,0000
0,0116	0,0014	0,0001	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,9999	0,8036	0,0000	0,0000
0,0235	0,0034	0,0003	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	1,0000	0,8741	0,0001	0,0000
0,0444	0,0076	0,0007	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	1,0000	0,9244	0,0003	0,0000
0,0780	0,0160	0,0019	0,0001	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	1,0000	0,9576	0,0008	0,0000
0,1279	0,0314	0,0045	0,0003	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	1,0000	0,9778	0,0021	0,0000
0,1966	0,0573	0,0100	0,0009	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	1,0000	0,9892	0,0049	0,0000
0,2840	0,0978	0,0207	0,0024	0,0001	0,0000	0,0000	0,0000	0,0000	0,0000	1,0000	0,9951	0,0108	0,0000
0,3866	0,1562	0,0396	0,0056	0,0004	0,0000	0,0000	0,0000	0,0000	0,0000	1,0000	0,9979	0,0222	0,0000
0,4981	0,2340	0,0710	0,0123	0,0010	0,0000	0,0000	0,0000	0,0000	0,0000	1,0000	0,9992	0,0424	0,0000
0,6100	0,3299	0,1187	0,0251	0,0026	0,0001	0,0000	0,0000	0,0000	0,0000	1,0000	0,9997	0,0756	0,0000
0,7138	0,4390	0,1861	0,0478	0,0063	0,0003	0,0000	0,0000	0,0000	0,0000	1,0000	0,9999	0,1259	0,0000
0,8026	0,5535	0,2736	0,0848	0,0139	0,0009	0,0000	0,0000	0,0000	0,0000	1,0000	1,0000	0,1964	0,0001
0,8727	0,6644	0,3784	0,1406	0,0287	0,0025	0,0001	0,0000	0,0000	0,0000	1,0000	1,0000	0,2874	0,0003
0,9235	0,7631	0,4940	0,2178	0,0551	0,0063	0,0002	0,0000	0,0000	0,0000	1,0000	1,0000	0,3954	0,0008
0,9573	0,8439	0,6111	0,3161	0,0983	0,0144	0,0007	0,0000	0,0000	0,0000	1,0000	1,0000	0,5132	0,0022
0,9780	0,9045	0,7199	0,4308	0,1631	0,0308	0,0019	0,0000	0,0000	0,0000	1,0000	1,0000	0,6310	0,0057
0,9896	0,9460	0,8122	0,5532	0,2519	0,0607	0,0053	0,0001	0,0000	0,0000	1,0000	1,0000	0,7388	0,0138
0,9955	0,9720	0,8837	0,6721	0,3630	0,1106	0,0132	0,0003	0,0000	0,0000	1,0000	1,0000	0,8285	0,0307
0,9982	0,9867	0,9339	0,7771	0,4890	0,1861	0,0301	0,0010	0,0000	0,0000	1,0000	1,0000	0,8965	0,0627
0,9994	0,9943	0,9658	0,8610	0,6184	0,2893	0,0628	0,0032	0,0000	0,0000	1,0000	1,0000	0,9430	0,1173
0,9998	0,9978	0,9840	0,9211	0,7378	0,4164	0,1199	0,0094	0,0000	0,0000	1,0000	1,0000	0,9716	0,2014
0,9999	0,9992	0,9933	0,9598	0,8363	0,5563	0,2089	0,0245	0,0002	0,0000	1,0000	1,0000	0,9873	0,3170
1,0000	0,9998	0,9975	0,9817	0,9084	0,6927	0,3319	0,0579	0,0008	0,0000	1,0000	1,0000	0,9950	0,4579
1,0000	0,9999	0,9992	0,9927	0,9547	0,8096	0,4812	0,1221	0,0032	0,0000	1,0000	1,0000	0,9983	0,6089
1,0000	1,0000	0,9998	0,9975	0,9806	0,8966	0,6387	0,2298	0,0118	0,0000	1,0000	1,0000	0,9995	0,7494
1,0000	1,0000	0,9999	0,9993	0,9930	0,9520	0,7806	0,3839	0,0378	0,0000	1,0000	1,0000	0,9999	0,8612
1,0000	1,0000	1,0000	0,9998	0,9979	0,9815	0,8879	0,5688	0,1036	0,0001	1,0000	1,0000	1,0000	0,9357
1,0000	1,0000	1,0000	1,0000	0,9995	0,9943	0,9540	0,7497	0,2396	0,0016	1,0000	1,0000	1,0000	0,9762
1,0000	1,0000	1,0000	1,0000	0,9999	0,9987	0,9858	0,8883	0,4595	0,0138	1,0000	1,0000	1,0000	0,9934
1,0000	1,0000	1,0000	1,0000	1,0000	0,9998	0,9971	0,9662	0,7206	0,0894	1,0000	1,0000	1,0000	0,9988
1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9997	0,9948	0,9231	0,3950	1,0000	1,0000	1,0000	0,9999
1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000

DE STANDAARD-NORMALE VERDELING

Verdelingsfunctie $\Phi(z) = P(Z \leq z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}t^2} dt$

z	0	1	2	3	4	5	6	7	8	9
−0,0	0,5000	0,4960	0,4920	0,4880	0,4840	0,4801	0,4761	0,4721	0,4681	0,4641
−0,1	0,4602	0,4562	0,4522	0,4483	0,4443	0,4404	0,4364	0,4325	0,4286	0,4247
−0,2	0,4207	0,4168	0,4129	0,4090	0,4052	0,4013	0,3974	0,3936	0,3897	0,3859
−0,3	0,3821	0,3783	0,3745	0,3707	0,3669	0,3632	0,3594	0,3557	0,3520	0,3483
−0,4	0,3446	0,3409	0,3372	0,3336	0,3300	0,3264	0,3228	0,3192	0,3156	0,3121
−0,5	0,3085	0,3050	0,3015	0,2981	0,2946	0,2912	0,2877	0,2843	0,2810	0,2776
−0,6	0,2743	0,2709	0,2676	0,2643	0,2611	0,2578	0,2546	0,2514	0,2483	0,2451
−0,7	0,2420	0,2389	0,2358	0,2327	0,2296	0,2266	0,2236	0,2206	0,2177	0,2148
−0,8	0,2119	0,2090	0,2061	0,2033	0,2005	0,1977	0,1949	0,1922	0,1894	0,1867
−0,9	0,1841	0,1814	0,1788	0,1762	0,1736	0,1711	0,1685	0,1660	0,1635	0,1611
−1,0	0,1587	0,1562	0,1539	0,1515	0,1492	0,1469	0,1446	0,1423	0,1401	0,1379
−1,1	0,1357	0,1335	0,1314	0,1292	0,1271	0,1251	0,1230	0,1210	0,1190	0,1170
−1,2	0,1151	0,1131	0,1112	0,1093	0,1075	0,1056	0,1038	0,1020	0,1003	0,0985
−1,3	0,0968	0,0951	0,0934	0,0918	0,0901	0,0885	0,0869	0,0853	0,0838	0,0823
−1,4	0,0808	0,0793	0,0778	0,0764	0,0749	0,0735	0,0721	0,0708	0,0694	0,0681
−1,5	0,0668	0,0655	0,0643	0,0630	0,0618	0,0606	0,0594	0,0582	0,0571	0,0559
−1,6	0,0548	0,0537	0,0526	0,0516	0,0505	0,0495	0,0485	0,0475	0,0465	0,0455
−1,7	0,0446	0,0436	0,0427	0,0418	0,0409	0,0401	0,0392	0,0384	0,0375	0,0367
−1,8	0,0359	0,0351	0,0344	0,0336	0,0329	0,0322	0,0314	0,0303	0,0301	0,0294
−1,9	0,0287	0,0281	0,0274	0,0268	0,0262	0,0256	0,0250	0,0244	0,0239	0,0233
−2,0	0,0228	0,0222	0,0217	0,0212	0,0207	0,0202	0,0197	0,0192	0,0188	0,0183
−2,1	0,0179	0,0174	0,0170	0,0166	0,0162	0,0158	0,0154	0,0150	0,0146	0,0143
−2,2	0,0139	0,0136	0,0132	0,0129	0,0125	0,0122	0,0119	0,0116	0,0113	0,0110
−2,3	0,0107	0,0104	0,0102	0,0099	0,0096	0,0094	0,0091	0,0089	0,0087	0,0084
−2,4	0,0082	0,0080	0,0078	0,0075	0,0073	0,0071	0,0069	0,0068	0,0066	0,0064
−2,5	0,0062	0,0060	0,0059	0,0057	0,0055	0,0054	0,0052	0,0051	0,0049	0,0048
−2,6	0,0047	0,0045	0,0044	0,0043	0,0041	0,0040	0,0039	0,0038	0,0037	0,0036
−2,7	0,0035	0,0034	0,0033	0,0032	0,0031	0,0030	0,0029	0,0028	0,0027	0,0026
−2,8	0,0026	0,0025	0,0024	0,0023	0,0023	0,0022	0,0021	0,0021	0,0020	0,0019
−2,9	0,0019	0,0018	0,0018	0,0017	0,0016	0,0016	0,0015	0,0015	0,0014	0,0014
−3,0	0,0013	0,0013	0,0013	0,0012	0,0012	0,0011	0,0011	0,0011	0,0010	0,0010
−3,1	0,0010	0,0009	0,0009	0,0009	0,0008	0,0008	0,0008	0,0008	0,0007	0,0007
−3,2	0,0007	0,0007	0,0006	0,0006	0,0006	0,0006	0,0006	0,0005	0,0005	0,0005
−3,3	0,0005	0,0005	0,0005	0,0004	0,0004	0,0004	0,0004	0,0004	0,0004	0,0003
−3,4	0,0003	0,0003	0,0003	0,0003	0,0003	0,0003	0,0003	0,0003	0,0003	0,0002
−3,5	0,0002	0,0002	0,0002	0,0002	0,0002	0,0002	0,0002	0,0002	0,0002	0,0002
−3,6	0,0002	0,0002	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001

DE STANDAARD-NORMALE VERDELING (vervolg)

z	0	1	2	3	4	5	6	7	8	9
0,0	0,5000	0,5040	0,5080	0,5120	0,5160	0,5199	0,5239	0,5279	0,5319	0,5359
0,1	0,5398	0,5438	0,5478	0,5517	0,5557	0,5596	0,5636	0,5675	0,5714	0,5753
0,2	0,5793	0,5832	0,5871	0,5910	0,5948	0,5987	0,6026	0,6064	0,6103	0,6141
0,3	0,6179	0,6217	0,6255	0,6293	0,6331	0,6368	0,6406	0,6443	0,6480	0,6517
0,4	0,6554	0,6591	0,6628	0,6664	0,6700	0,6736	0,6772	0,6808	0,6844	0,6879
0,5	0,6915	0,6950	0,6985	0,7019	0,7054	0,7088	0,7123	0,7157	0,7190	0,7224
0,6	0,7257	0,7291	0,7324	0,7357	0,7389	0,7422	0,7454	0,7486	0,7517	0,7549
0,7	0,7580	0,7611	0,7642	0,7673	0,7704	0,7734	0,7764	0,7794	0,7823	0,7852
0,8	0,7881	0,7910	0,7939	0,7967	0,7995	0,8023	0,8051	0,8078	0,8106	0,8133
0,9	0,8159	0,8186	0,8212	0,8238	0,8264	0,8289	0,8315	0,8340	0,8365	0,8389
1,0	0,8413	0,8438	0,8461	0,8485	0,8508	0,8531	0,8554	0,8577	0,8599	0,8621
1,1	0,8643	0,8665	0,8686	0,8708	0,8729	0,8749	0,8770	0,8790	0,8810	0,8830
1,2	0,8849	0,8869	0,8888	0,8907	0,8925	0,8944	0,8962	0,8980	0,8997	0,9015
1,3	0,9032	0,9049	0,9066	0,9082	0,9099	0,9115	0,9131	0,9147	0,9162	0,9177
1,4	0,9192	0,9207	0,9222	0,9236	0,9251	0,9265	0,9279	0,9292	0,9306	0,9319
1,5	0,9332	0,9345	0,9357	0,9370	0,9382	0,9394	0,9406	0,9418	0,9429	0,9441
1,6	0,9452	0,9463	0,9474	0,9484	0,9495	0,9505	0,9515	0,9525	0,9535	0,9545
1,7	0,9554	0,9564	0,9573	0,9582	0,9591	0,9599	0,9608	0,9616	0,9625	0,9633
1,8	0,9641	0,9649	0,9656	0,9664	0,9671	0,9678	0,9686	0,9693	0,9699	0,9706
1,9	0,9713	0,9719	0,9726	0,9732	0,9738	0,9744	0,9750	0,9756	0,9761	0,9767
2,0	0,9772	0,9778	0,9783	0,9788	0,9793	0,9798	0,9803	0,9808	0,9812	0,9817
2,1	0,9821	0,9826	0,9830	0,9834	0,9838	0,9842	0,9846	0,9850	0,9854	0,9857
2,2	0,9861	0,9864	0,9868	0,9871	0,9875	0,9878	0,9881	0,9884	0,9887	0,9890
2,3	0,9893	0,9896	0,9898	0,9901	0,9904	0,9906	0,9909	0,9911	0,9913	0,9916
2,4	0,9918	0,9920	0,9922	0,9925	0,9927	0,9929	0,9931	0,9932	0,9934	0,9936
2,5	0,9938	0,9940	0,9941	0,9943	0,9945	0,9946	0,9948	0,9949	0,9951	0,9952
2,6	0,9953	0,9955	0,9956	0,9957	0,9959	0,9960	0,9961	0,9962	0,9963	0,9964
2,7	0,9965	0,9966	0,9967	0,9968	0,9969	0,9970	0,9971	0,9972	0,9973	0,9974
2,8	0,9974	0,9975	0,9976	0,9977	0,9977	0,9978	0,9979	0,9979	0,9980	0,9981
2,9	0,9981	0,9982	0,9982	0,9983	0,9984	0,9984	0,9985	0,9985	0,9986	0,9986
3,0	0,9987	0,9987	0,9987	0,9988	0,9988	0,9989	0,9989	0,9989	0,9990	0,9990
3,1	0,9990	0,9991	0,9991	0,9991	0,9992	0,9992	0,9992	0,9992	0,9993	0,9993
3,2	0,9993	0,9993	0,9994	0,9994	0,9994	0,9994	0,9994	0,9995	0,9995	0,9995
3,3	0,9995	0,9995	0,9995	0,9996	0,9996	0,9996	0,9996	0,9996	0,9996	0,9997
3,4	0,9997	0,9997	0,9997	0,9997	0,9997	0,9997	0,9997	0,9997	0,9997	0,9998
3,5	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998
3,6	0,9998	0,9998	0,9999	0,9999	0,9999	0,9999	0,9999	0,9999	0,9999	0,9999

Overschrijdingskansen (één-, resp. tweezijdig)

z	0,675	1,282	1,645	1,960	2,326	2,576	3,090
$P(Z > z)$	0,25	0,10	0,05	0,025	0,01	0,005	0,001
$P(\|Z\| > z)$	0,50	0,20	0,10	0,05	0,02	0,01	0,002