CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam “Wiskunde B”

Date: 16 January 2014

Time: 14.00 – 17.00

Please read the instructions below carefully before answering the questions.

- This exam consists of 5 questions, with in total 16 sub-questions.
- Points that can be scored:

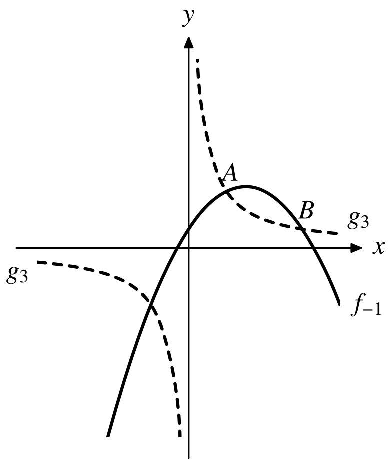
question	1	2	3	4	5
a	3	3	5	6	6
b	6	6	3	6	6
c	7		5	6	8
d	6		8		
total	22	9	21	18	20

  You will pass the exam if you score a total of at least 45 points out of a possible 90 points.
- Make sure your name is clearly written on every answer sheet.
- Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your (graphing) calculator are invalid.
- Write your answers in ink. Do not use a pencil, except when drawing graphs.
- You can use a (graphing) calculator. The use of hand-held computers is not allowed. If there is doubt about the status of your equipment, the exam monitor will decide whether it is allowed for use during the exam.
- On page 5 you will find formulas and definitions that you may use during this exam. The use of other formula sheets or books (like BINAS) is not allowed.
- You can use a dictionary if it is approved by the exam monitor.
- Please switch off your mobile telephone.
- Please check www.ccvx.nl for further information on this exam (unfortunately in Dutch only). Answers to the questions will be published on this website next week. Do not call the Open Universiteit, since they do not have any further information about this exam.

1 For $x \neq 0$ the functions f_a and g_b are given by

$$f_a(x) = ax^2 + 3x + 1 \text{ and } g_b(x) = \frac{b}{x}$$

The graphs of f_{-1} and g_3 are shown in the figure below.



As you can see, the graphs of f_{-1} and g_3 intersect in three points. Two of these intersections, A and B, ae to the right of the y-axis.

3 pt **a** Show that $x_A = 1$ and $x_B = 3$.

6 pt **b** Compute exactly the area of the region enclosed by the graphs of f_{-1} and g_3.

The function h is given by $h(x) = f_{\frac{1}{2}}(x) + g_4(x)$.

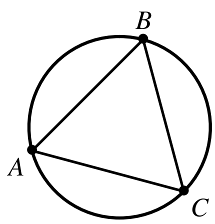
7 pt **c** Show algebraically that the graph of h has one point of inflection.

6 pt **d** Compute exactly for which values of a and b the graphs of f_a and g_b are tangent to each other in a point with x-coordinate 1.

2 Given a triangle ABC with $\angle B = \angle C$.
D is the intersection of the perpendicular from A with side BC.

3 pt **a** Show that the triangles ABD and ACD are congruent.

In the figure to the right an equilateral triangle ABC and it's circumscribed circle are shown.
The intersection of the perpendiculars from the angles of this triangle is called H.
Point P is on the shortest arc AB.
On page 4 you will find two enlarged copies of this figure.



6 pt **b** Show that $\angle APB = \angle AHB$.

3 Given are the of functions $f(x) = \ln(x + \mathrm{e})$ and $g(x) = \ln(x + 2\mathrm{e})$.
A vertical line $x = p$ intersects the graph of f in point P and the graph of g in point Q.
The distance between P and Q equals ln(3).

5 pt **a** Compute the value of p exactly.

3 pt **b** Show that the function $F(x) = x \cdot \ln(x + \mathrm{e}) + \mathrm{e} \cdot \ln(x + \mathrm{e}) - x$ is a primitive function (anti-derivative) of f.

V is the region enclosed by the negative x-axis, the positive y-axis and the graph of f.

5 pt **c** Compute the area of V algebraically.

8 pt **d** Compute algebraically the volume of the figure that is created by rotating V around the y-axes.

4 Given the function $f(x) = 2^{x+1} + 2^{4-x}$.

6 pt **a** Solve exactly: $f(x) = 33$.

6 pt **b** Compute exactly the value of a for which the horizontal line $y = a$ is a tangent to the graph of f.
Write your answer in the form $a = 2^b$.

There is a value of a for which the line $y = a$ intersects the graph of f in two points P and Q. The length of the line segment PQ equals 4.

6 pt **c** Compute this value of a exactly and write the answer in the form $a = b\sqrt{2}$.

5 Given the functions $f(x) = \sin(x) + \cos(3x)$ en $g(x) = \sin(x) \cdot \sin(2x)$.

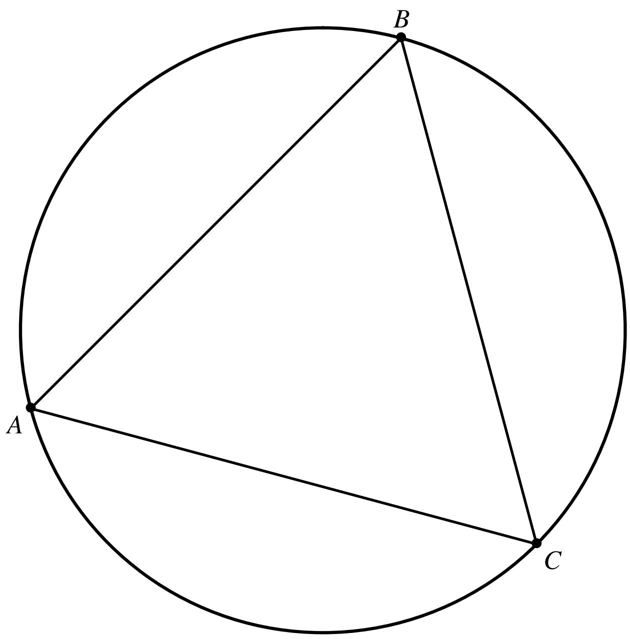
6 pt **a** Solve exactly: $f(x) = 0$.

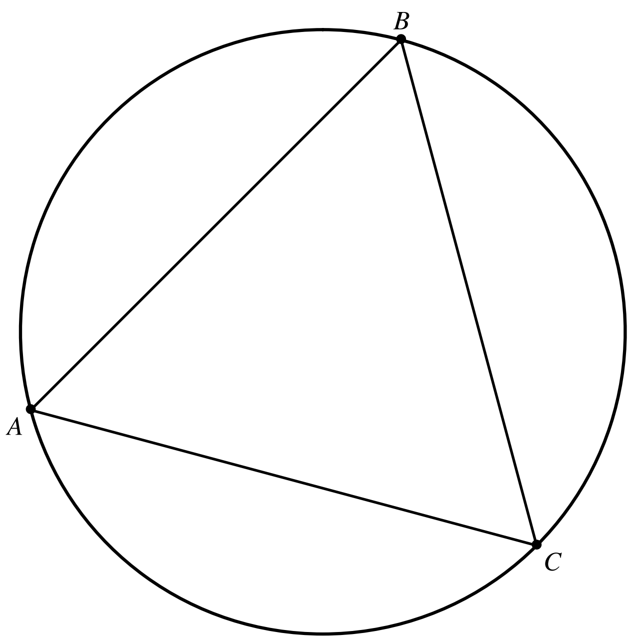
6 pt **b** Solve exactly: $g(x) = \cos(x)$.

The graph of g has three horizontal tangents.

8 pt **c** Compute the equations of these three horizontal tangents exactly.

Answer sheet for question 2   Name: ________________





Formulas and definitions you may use in the exam Wiskunde B

Geometry

References to plane geometry theorems and definitions used in a proof may be used without further explanation. Translation of the official list on the Dutch version of this exam.

Angles, lines and distances:
straight angle, right angle, opposite angles, F-angles, Z-angles, distance point to line, triangle inequality.

Loci:
perpendicular middle line, bisector, pair of bisectors, middle parallel, circle, parabola.

Triangles:
sum of angles of a triangle, outside angle of a triangle
Cases of congruent triangels: ASA, SAA, SAS, SSS, SSP
(A = angle; S = side; P = perpendicular angle (90°))
Cases of similar triangles: aa, sas, sss, ssp
perpendicular middle lines of a triangle, angle bisectors of a triangle (definition and theorem), perpendiculars from an angle (definition and theorem), medians (definition and theorem), isosceles triangle, equilateral triangle, right-angled triangle, Pythagoras, isosceles right-angled triangle, half equilateral triangle.

Quadrilaterals:
sum of angles of a quadrilateral, parallelogram, rhombus, rectangle, square.

Circle, chords, arcs, angles, tangent line, quadrilaterals:
chord, arc and chord, perpendicular line to chord, centerline, Thales, central angle, inscribed angle, constant angle, tangent, angle between chord and tangent, cyclic quadrilateral

Trigonometry

$$\sin(t+u) = \sin t \cos u + \cos t \sin u$$

$$\sin t + \sin u = 2 \sin \frac{t+u}{2} \cos \frac{t-u}{2}$$

$$\sin(t-u) = \sin t \cos u - \cos t \sin u$$

$$\sin t - \sin u = 2 \sin \frac{t-u}{2} \cos \frac{t+u}{2}$$

$$\cos(t+u) = \cos t \cos u - \sin t \sin u$$

$$\cos t + \cos u = 2 \cos \frac{t+u}{2} \cos \frac{t-u}{2}$$

$$\cos(t-u) = \cos t \cos u + \sin t \sin u$$

$$\cos t - \cos u = -2 \sin \frac{t+u}{2} \sin \frac{t-u}{2}$$