**CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE**

# Entrance Exam "Wiskunde B"

Date: 16 January 2015
Time: 13.30 – 16.30
Questions: 5

**Please read the instructions below carefully before answering the questions.**

There is a preprinted answer sheet for question 2b.

Make sure your name is clearly written on each answer sheet.

Take a new answer sheet for each question.

Show all your calculations clearly. Illegible answers and answers without a calculation of an explanation of the use of your graphing calculator are invalid.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use Tipp-ex.

You can use a (graphing) calculator. The use of hand-held computers is not allowed. If there is doubt about the status of your equipment, the invigilator will decide whether it is allowed for use during the exam.

On the last page of this exam you will find formulas and definitions that you may use during this exam. The use of other formula sheets or books (like BINAS) is not allowed.

You can use a dictionary if it is approved by the invigilator.

During the exam, the use of a mobile telephone or other electrical equipment is not allowed. Please switch off your mobile telephone.

Please check www.ccvx.nl for more information on this exam (unfortunately most information is available in Dutch only). Answers to the questions of this exam will be published on this website early next week.

Points that can be scored for each sub-question:

| Question | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| a | 7 | 4 | 6 | 6 | 4 |
| b | 6 | 5 | 7 | 5 | 6 |
| c | 6 | | 7 | 4 | 7 |
| d | | | | 4 | 6 |
| Total | 19 | 9 | 20 | 19 | 23 |

$$\text{Grade} = \frac{\text{total points scored}}{10} + 1$$

You will pass the exam if your grade is at least 5.5

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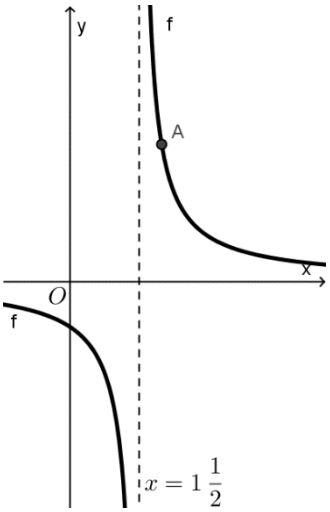
## Question 1

For $x \neq 1\frac{1}{2}$ the function $f$ is given by

$$f(x) = \frac{3}{2x - 3}$$

The figure to the right shows a sketch of the graph of this function.
$A$ is the point (2,3).




The tangent to the graph of $f$ in point $A$ intersects the $x$-axis in point $P$ and intersects the $y$-axis in point $Q$.

7pt a Compute the area of triangle $OPQ$ algebraically.

For all $q > 2$, $V_q$ is the region enclosed by the graph of $f$, the line $x = 2$, the line $x = q$ and the $x$-axis.

6pt b Compute algebraically the value of $q$ for which the area of region $V_q$ equals 3.

There is one straight line trough $O(0,0)$ that is a tangent to the graph of $f$.

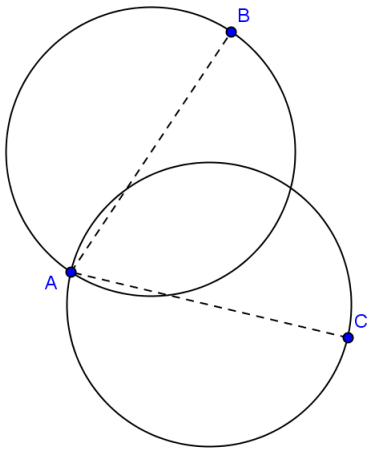
6pt c Find an equation for this line algebraically.

## Question 2

Given a triangle $ABC$ with $\angle A = \angle B$ and $\angle C < 90°$.
$M$ is the center of the circumscribed circle of this triangle

4pt a Show that $CM$ is the bisector of $\angle C$.

In the figure to the right, $A$, $B$ and $C$ are the vertices of a triangle. De circle with diameter $AB$ and the circle with diameter $AC$ intersect not only in point $A$, but also in a second point.
*On the answer sheet you will find an enlarged copy of this figure.*




5pt b Show that this second intersection of these circles is on edge $BC$ of the triangle.

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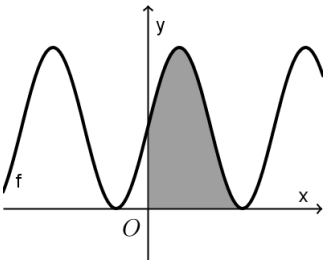
## Question 3




In the figure to the right a part of the graph is shown for the function

$$f(x) = 4\sin(2x)\cos(2x) + 2$$

It is suggested that the points where $f$ has a minimum are all situated on the *x*-axes.

6pt a Show algebraically that this is indeed the case.

The shaded region is enclosed by the graph of *f*, the *x*-axis and the *y*-axis.

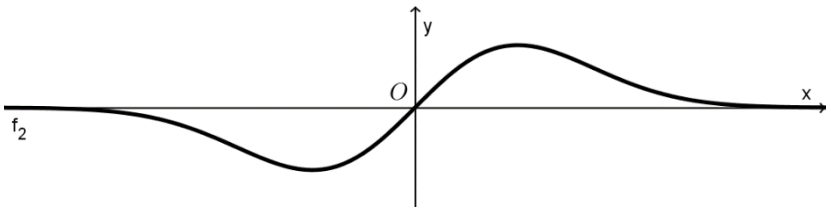
7pt b Compute the area of this region algebraically.

The function $g$ is given by $g(x) = 4\sin(3x)\sin(2x) + 2$.

7pt c Solve the equation $f(x) = g(x)$ algebraically and find all solutions in the interval $[0\,,\pi]$.

## Question 4

For each $a > 0$ the function $f_a$ is given by $f_a(x) = x \cdot \mathrm{e}^{-ax^2}$.
In the figure below a sketch of the graph of $f_a$ is shown for $a = 2$.




6pt a Compute the range of the function $f_2$ algebraically.

5pt b Show algebraically that the turning points (minimums and maximums) of the graphs of the functions $f_a$ are all on the same straight line and find the equation of this line.

In the figure you can see that the graph of $f_2$ has three points of inflection.

4pt c Compute the *x*-coordinates of these three points of inflexion algebraically.

4pt d Find the values of $p$ for which the line $y = px$ has three intersections with the graph of $f_2$.

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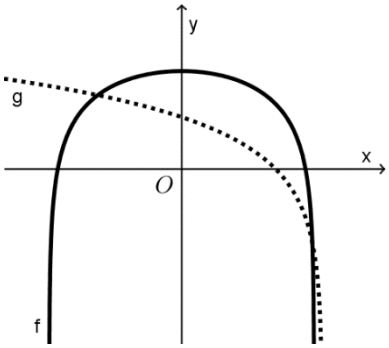
## Question 5




To the right the graphs are shown of the functions $f(x) = \ln(8 - x^2)$ and $g(x) = \ln(3 - x)$.

The function *h* is given by $h(x) = f(x) - g(x)$.

4pt a Compute the *x*-coordinates of the graphs of *f* and *g* algebraically.

6pt b Solve algebraically: $h(x) = \ln\left(\frac{4}{5}\right)$.

7pt c Compute algebraically the value(s) of *x* for which *h* has an extreme value.

*V* is the region enclosed by the graph of *f* and the *x*-axis.

6pt d Compute exactly the volume of the figure that is created by rotating *V* around the *y*-axis.

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# Formulas and definitions you may use in the exam Wiskunde B

## Geometry

*References to plane geometry theorems and definitions used in a proof may be used without further explanation. Translation of the official list on the Dutch version of this exam.*

Angles, lines and distances:
straight angle, right angle, opposite angles, F-angles, Z-angles, distance point to line, triangle inequality.

Loci:
perpendicular middle line, bisector, pair of bisectors, middle parallel, circle, parabola.

Triangles:
sum of angles of a triangle, outside angle of a triangle
Cases of congruent triangles: ASA, SAA, SAS, SSS, SSP
(A = angle; S = side; P = perpendicular angle (90º))
Cases of similar triangles: aa, sas, sss, ssp
perpendicular middle lines of a triangle, angle bisectors of a triangle (definition and theorem), perpendiculars from an angle (definition and theorem), medians (definition and theorem), isosceles triangle, equilateral triangle, right-angled triangle, Pythagoras, isosceles right-angled triangle, half equilateral triangle.

Quadrilaterals:
sum of angles of a quadrilateral, parallelogram, rhombus, rectangle, square.

Circle, chords, arcs, angles, tangent line, quadrilaterals:
chord, arc and chord, perpendicular line to chord, centerline, Thales, central angle, inscribed angle, constant angle, tangent, angle between chord and tangent, cyclic quadrilateral

## Trigonometry

$\sin(t+u) = \sin t \cos u + \cos t \sin u$

$\sin(t-u) = \sin t \cos u - \cos t \sin u$

$\cos(t+u) = \cos t \cos u - \sin t \sin u$

$\cos(t-u) = \cos t \cos u + \sin t \sin u$

$\sin t + \sin u = 2 \sin \frac{t+u}{2} \cos \frac{t-u}{2}$

$\sin t - \sin u = 2 \sin \frac{t-u}{2} \cos \frac{t+u}{2}$

$\cos t + \cos u = 2 \cos \frac{t+u}{2} \cos \frac{t-u}{2}$

$\cos t - \cos u = -2 \sin \frac{t+u}{2} \sin \frac{t-u}{2}$

*End of the exam.*

*Is your name on all answer sheets?*

*If necessary, the invigilator can provide an extra copy of the preprinted answer sheet for question 2b.*

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