CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam "Wiskunde B"

Date: 28 July 2014

Time: 14.00 – 17.00

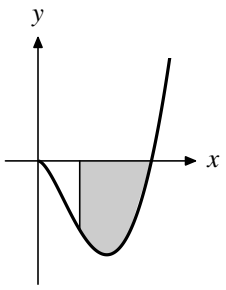
Please read the instructions below carefully before answering the questions.

- This exam consists of 5 questions, with in total 18 sub-questions.
- Points that can be scored:

question	1	2	3	4	5
a	6	4	5	6	4
b	3	3	4	7	3
c	6	3	6	6	6
d			6	4	8
total	15	10	21	23	21

  You will pass the exam if you score a total of at least 45 points out of a possible 90 points.
- Make sure your name is clearly written on every answer sheet.
- Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your (graphing) calculator are invalid.
- Write your answers in ink. Do not use a pencil, except when drawing graphs.
- You can use a (graphing) calculator. The use of hand-held computers is not allowed. If there is doubt about the status of your equipment, the exam monitor will decide whether it is allowed for use during the exam.
- On page 5 you will find formulas and definitions that you may use during this exam. The use of other formula sheets or books (like BINAS) is not allowed.
- You can use a dictionary if it is approved by the exam monitor.
- Please switch off your mobile telephone.
- Please check www.ccvx.nl for further information on this exam (unfortunately in Dutch only). Answers to the questions will be published on this website next week. Do not call the Open Universiteit, since they do not have any further information about this exam.

1 On the right, the graph is shown of the function $f(x) = 9x^2 \cdot \ln x$.



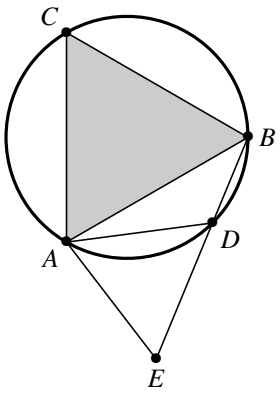
6 pt **a** Compute exactly the coordinates of the point of inflexion of the graph of f.

3 pt **b** Show that $F(x) = x^3 \cdot (3 \ln x - 1)$ is a primitive function (= an antiderivative) of f.

In the figure, the shaded region is enclosed by the x-axis, the graph of f and the line $x = \frac{1}{e}$.

6 pt **c** Compute exactly the area of this region.

2 The points A, B and C in the figure to the right are the vertices of an equilateral triangle.
Point D is on the circle trough A, B and C, on the arc between A and B.
Point E is on the extension of the line segment BD, with $|AE| = |DE|$.
On page 4 you will find an enlarged version of this figure.



4 pt **a** Show that ADE is an equilateral triangle.

3 pt **b** Show that CAD and BAE are congruent triangles.

3 pt **c** Show that $|DC| = |DA| + |DB|$

3 Given are the functions $f_a(x) = \frac{\sqrt{x}}{x + a}$ and $g(x) = \sqrt{x} - \frac{2}{\sqrt{x}}$.

5 pt **a** Compute the values of a for which the graph of f_a has one horizontal tangent.

4 pt **b** Find the range of f_1.
Do not use the graphing calculator in the explanation of your answer.

6 pt **c** Solve algebraically: $f_1(x) = g(x)$.

V is the region enclosed by the graph of g, the graph of $y = \sqrt{x}$ and the lines $x = 1$ and $x = 4$.

6 pt **d** Compute the area of V algebraically.

4 Given are the functions $f(x) = \cos(x) + \cos(\frac{1}{3}\pi - x)$ and $g(x) = \sin(x) + \sin(\frac{1}{3}\pi - x)$.
A is the point on the graph of f for which $x_A = -\frac{1}{6}\pi$.

6 pt **a** Compute exactly an equation for the tangent tot the graph of f in point A.

7 pt **b** Solve the equation $g(x) = 0$ algebraically and give all solutions in the interval $[-2\pi,2\pi]$

The function h is given by $h(x) = \dfrac{f(x)}{g(x)}$.

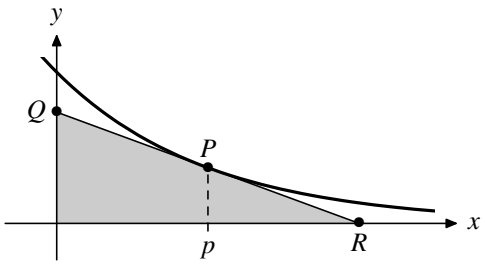
6 pt **c** Show algebraically that $h'(x) = 0$ for all x in the domain of h.

4 pt **d** Use the result of question c to describe the graph of h.

5 Given the function $f(x) = \mathrm{e}^{-x}$.
$P\,(p,f(p))$ is some point on the graph of f.

4 pt **a** Show that the equation of the tangent to the graph of f in point P can be written as $y = \mathrm{e}^{-p} \cdot (1 + p - x)$.

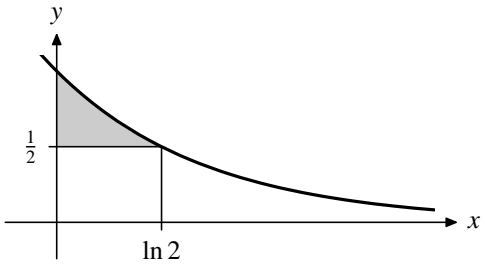
For $p > 0$ the tangent to the graph of f in point P intersects the y-axis in point Q. R is the intersection of this tangent with the x-axis. In this way, the triangle OQR is formed (the shaded region in the figure below).



3 pt **b** Show that for each $p > 0$ the area of triangle OQR is equal to $\dfrac{(p+1)^2}{2} \cdot \mathrm{e}^{-p}$.

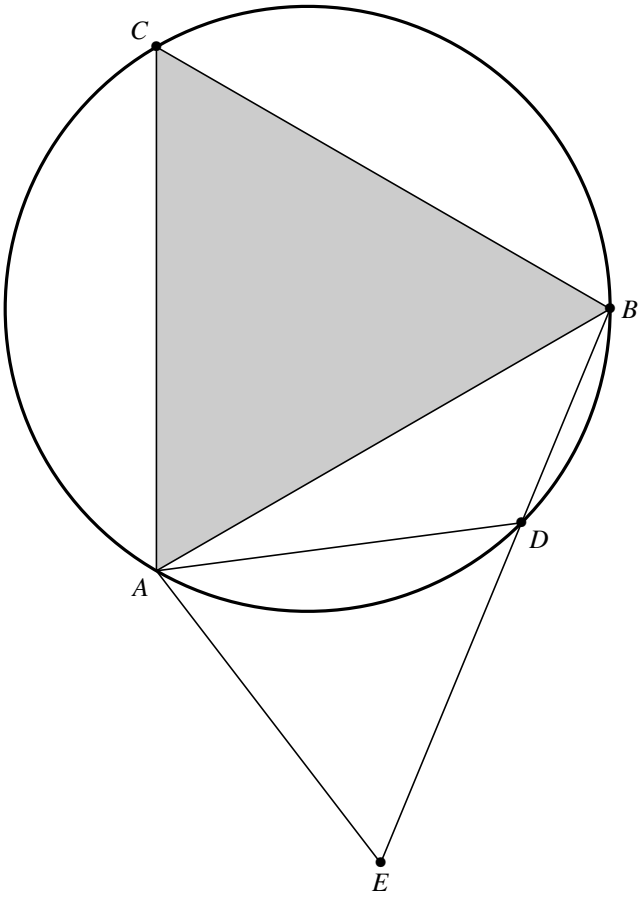
6 pt **c** Compute exactly the value of p for which the area of triangle OQR is maximal.

V is the region enclosed by the graph of f, the horizontal line $y = \frac{1}{2}$ and the y-axis (the shaded region in the figure below).



8 pt **d** Compute exactly the volume of the figure that is created by rotating V around the x-axis.

Answer sheet for question 2 Name: _____



Formulas and definitions you may use in the exam Wiskunde B

Geometry

References to plane geometry theorems and definitions used in a proof may be used without further explanation. Translation of the official list on the Dutch version of this exam.

Angles, lines and distances:
straight angle, right angle, opposite angles, F-angles, Z-angles, distance point to line, triangle inequality.

Loci:
perpendicular middle line, bisector, pair of bisectors, middle parallel, circle, parabola.

Triangles:
sum of angles of a triangle, outside angle of a triangle
Cases of congruent triangels: ASA, SAA, SAS, SSS, SSP
(A = angle; S = side; P = perpendicular angle (90°))
Cases of similar triangles: aa, sas, sss, ssp
perpendicular middle lines of a triangle, angle bisectors of a triangle (definition and theorem), perpendiculars from an angle (definition and theorem), medians (definition and theorem), isosceles triangle, equilateral triangle, right-angled triangle, Pythagoras, isosceles right-angled triangle, half equilateral triangle.

Quadrilaterals:
sum of angles of a quadrilateral, parallelogram, rhombus, rectangle, square.

Circle, chords, arcs, angles, tangent line, quadrilaterals:
chord, arc and chord, perpendicular line to chord, centerline, Thales, central angle, inscribed angle, constant angle, tangent, angle between chord and tangent, cyclic quadrilateral

Trigonometry

$$\sin(t+u) = \sin t \cos u + \cos t \sin u \qquad \sin t + \sin u = 2 \sin \frac{t+u}{2} \cos \frac{t-u}{2}$$

$$\sin(t-u) = \sin t \cos u - \cos t \sin u \qquad \sin t - \sin u = 2 \sin \frac{t-u}{2} \cos \frac{t+u}{2}$$

$$\cos(t+u) = \cos t \cos u - \sin t \sin u \qquad \cos t + \cos u = 2 \cos \frac{t+u}{2} \cos \frac{t-u}{2}$$

$$\cos(t-u) = \cos t \cos u + \sin t \sin u \qquad \cos t - \cos u = -2 \sin \frac{t+u}{2} \sin \frac{t-u}{2}$$