CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam "Wiskunde B"

Date: 3 June 2014

Time: 14.00 – 17.00

Please read the instructions below carefully before answering the questions.

- This exam consists of 5 questions, with in total 15 sub-questions.
- Points that can be scored:

question	1	2	3	4	5
a	5	5	6	6	5
b	8	4	8	6	8
c	7		4	6	6
d					6
total	20	9	18	18	25

  You will pass the exam if you score a total of at least 45 points out of a possible 90 points.
- Make sure your name is clearly written on every answer sheet.
- Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your (graphing) calculator are invalid.
- Write your answers in ink. Do not use a pencil, except when drawing graphs.
- You can use a (graphing) calculator. The use of hand-held computers is not allowed. If there is doubt about the status of your equipment, the exam monitor will decide whether it is allowed for use during the exam.
- On page 4 you will find formulas and definitions that you may use during this exam. The use of other formula sheets or books (like BINAS) is not allowed.
- You can use a dictionary if it is approved by the exam monitor.
- Please switch off your mobile telephone.
- Please check www.ccvx.nl for further information on this exam (unfortunately in Dutch only). Answers to the questions will be published on this website next week. Do not call the Open Universiteit, since they do not have any further information about this exam.

1 Given is the function $f(x) = 8x^{\frac{3}{2}}$.

Point A is the point on the graph of f for which $x_A = 4$.
Point B is the intersection of the y-axis and the tangent to the graph of f in point A.

5 pt **a** Compute y_B algebraically.

V is the region enclosed by the graph of f, the y-axis and the line $y = 1$.

8 pt **b** Compute the area of region V algebraically.

There are two tangents to the graph of f which pass trough the point (1,0).

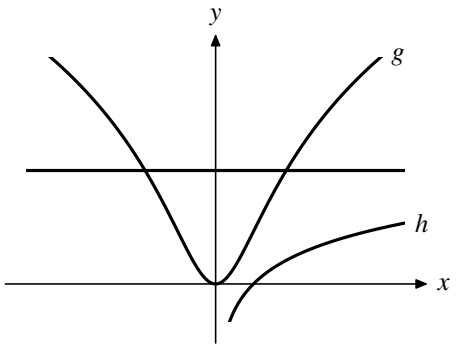
7 pt **c** Compute algebraically the coordinates of the points where these tangent touch the graph of f.

2 $ABCD$ is a quadrilateral for which the sides AB and CD are parallel and $\angle A$ and $\angle B$ are both acute.

5 pt **a** Show that if sides AD and BC have the same length, the vertices of this quadrilateral are on one circle.

4 pt **b** Show that if the vertices of this quadrilateral are on one circle, the sides AD and BC have the same length.

3 In figure below the graphs are shown of the function $g(x) = 2\ln(x^2 + 2)$, the horizontal line $y = 3$ and the function $h(x) = \ln x$.



The graph of g has two points of inflexion.

6 pt **a** Compute the coordinates of these two points of inflexion algebraically.

V is the region enclosed by the graph of g and the line $y = 3$.

8 pt **b** Compute algebraically the volume of the figure that is created by rotating V around the y-axes.

For each $p > 0$, A_p is the point $(p,g(p))$ and B_p is the point $(p,h(p))$.
Since the graphs of g and h do not intersect, there is a value of p for which the distance between the points A_p and B_p is minimal.

4 pt **c** Compute this value of p algebraically.

4 On the interval $[0,\pi]$ the function k is given by $k(x) = \sin 2x + \cos 2x$.
Point P is the point on the graph of k for which $x_P = \frac{1}{4}\pi$.
The tangent to the graph of k in point P intersects the x-axis in point Q.

6 pt **a** Compute exactly the area of triangle OPQ.

6 pt **b** Compute exactly the area of the region enclosed by the graph of k, the x-axis and the y-axis.

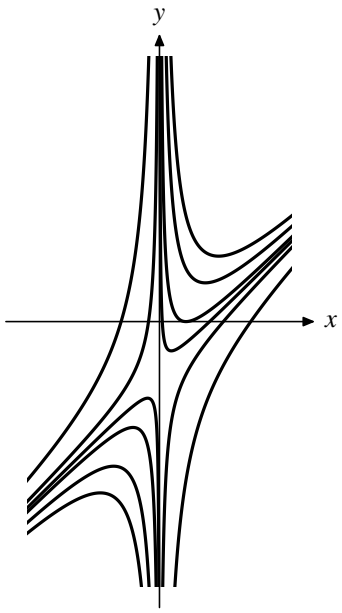
The graph of k is a sinusoide (you do not have to show this).
Therefore, $k(x)$ can be written in the form $k(x) = a + b \cdot \sin(cx + d)$

6 pt **c** Compute the exact values of a, b, c and d.

5 For all real values of a the function f_a is given by

$$f_a(x) = x - 2 + \frac{a-3}{x}$$

In the figure to the right the graphs of f_a are shown for a number of values of a.



5 pt **a** Compute exactly for which values of a the graph of f_a has two *different* intersections with the x-axis.

In the figure we can see that for some values of a, f_a has two extremes: there is a highest point to the left of the x-axis and there is a lowest point to the right of the x-axis. For other values of a, f_a does not have any extremes.

8 pt **b** Compute exactly for which values of a the function f_a has extremes and show that the highest and lowest points of the graphs of those functions are all on the same straight line.

6 pt **c** Compute exactly the area of the region enclosed by the horizontal line $y = 1$ and the graph of f_5.

6 pt **d** Compute exactly the value(s) of q for which the horizontal line $y = q$ intersects the graph of f_8 in two points that are located on a distance 4 from each other.

Formulas and definitions you may use in the exam Wiskunde B

Geometry

References to plane geometry theorems and definitions used in a proof may be used without further explanation. Translation of the official list on the Dutch version of this exam.

Angles, lines and distances:
straight angle, right angle, opposite angles, F-angles, Z-angles, distance point to line, triangle inequality.

Loci:
perpendicular middle line, bisector, pair of bisectors, middle parallel, circle, parabola.

Triangles:
sum of angles of a triangle, outside angle of a triangle
Cases of congruent triangels: ASA, SAA, SAS, SSS, SSP
(A = angle; S = side; P = perpendicular angle (90°))
Cases of similar triangles: aa, sas, sss, ssp
perpendicular middle lines of a triangle, angle bisectors of a triangle (definition and theorem), perpendiculars from an angle (definition and theorem), medians (definition and theorem), isosceles triangle, equilateral triangle, right-angled triangle, Pythagoras, isosceles right-angled triangle, half equilateral triangle.

Quadrilaterals:
sum of angles of a quadrilateral, parallelogram, rhombus, rectangle, square.

Circle, chords, arcs, angles, tangent line, quadrilaterals:
chord, arc and chord, perpendicular line to chord, centerline, Thales, central angle, inscribed angle, constant angle, tangent, angle between chord and tangent, cyclic quadrilateral

Trigonometry

$$\sin(t+u) = \sin t \cos u + \cos t \sin u \qquad \sin t + \sin u = 2 \sin \frac{t+u}{2} \cos \frac{t-u}{2}$$

$$\sin(t-u) = \sin t \cos u - \cos t \sin u \qquad \sin t - \sin u = 2 \sin \frac{t-u}{2} \cos \frac{t+u}{2}$$

$$\cos(t+u) = \cos t \cos u - \sin t \sin u \qquad \cos t + \cos u = 2 \cos \frac{t+u}{2} \cos \frac{t-u}{2}$$

$$\cos(t-u) = \cos t \cos u + \sin t \sin u \qquad \cos t - \cos u = -2 \sin \frac{t+u}{2} \sin \frac{t-u}{2}$$